Exercise

Let $M_{n,k}$ be the number of $\{(1,1),(1,-1)\}$-walks in $\mathbb{N}^2$ of length n that start at $(0,0)$ and end at vertical altitude k. Let $M(x,y) = \sum_{n,k} M_{n,k} x^n y^k$.

(a) Show that $(y - x(1+y^2)) \cdot M(x,y) = y - x \cdot M(x,0)$

(b) Deduce that $M(x,y) = \dfrac{\sqrt{1-4x^2} + 2xy - 1}{2x(y - x(1+y^2))}$