Alternative to Gaussian elimination

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Overview

Matrix polynomials

Minimal bases

New elimination
Matrix polynomials
A \in K[x]^{n \times n} \text{ of degree } d

\det A(x) = \sum_{\sigma} \prod_{i=1}^{n} A_{\sigma(i),i}

\deg \det A \leq nd
$A \in \mathbb{Z}[x]^{n \times n}$ with entries of absolute values less than $b$

$$\det A = \prod_{j=1}^{n} ||A_j^*|| \leq \prod_{j=1}^{n} ||A_j|| \leq b^n n^{n/2}$$

For input size $\beta$:

$$\log \det A \leq n\beta + O(n \log n)$$
Hadamard's conjecture

\[ a_{i,j} \in \{1, -1\} \text{ and the rows of } A \text{ are mutually orthogonal} \]

A Hadamard matrix of dimension \( n \) exists for every \( n \) multiple of 4