Two exercises for next time (8/2/2021)

1. Compute a telescoper for the diagonal of the rational power series

\[
\frac{1}{1 - x - y} = \sum_{i,j \geq 0} \binom{i + j}{i} x^i y^j
\]

in two different ways:

1. using the 2G (Almkvist-Zeilberger) creative telescoping algorithm;
2. using the 4G (Hermite reduction-based) creative telescoping algorithm.

2. Let \( f, g \in \mathbb{Q}[x] \) be two coprime polynomials. Let \( h \in \mathbb{Q}[x] \) be another polynomial such that \( \deg h < \deg f + \deg g \).

1. Show that the equation

\[
sf + tg = h
\]

admits an unique solution \( (s, t) \in \mathbb{Q}[x]^2 \) s.t. \( \deg s < \deg g, \quad \deg t < \deg f \).
2. Design an algorithm for computing the solution \( (s, t) \) starting from \( (f, g, h) \) in quasi-optimal complexity.