Exercises on the chapters
“Padé approximants” and “Sparse matrices”

To prepare for 2020-11-02

**Exercise 1.** Prove algorithmically that there is no Padé approximant of type $(1,1)$ for $1 + x^2$, i.e. no pair $(R,V)$ of polynomials in $\mathbb{Q}[x]$ of degree at most 1 such that $V(0) \neq 0$ and $\frac{R}{V} = 1 + x^2 \mod x^3$.

**Exercise 2.** Let $\mathbb{K} = \mathbb{Z}/5\mathbb{Z}$ be the finite field with 5 elements, let $M \in \mathcal{M}_3(\mathbb{K})$ and $b \in \mathbb{K}^3$ be

$$M = \begin{pmatrix} 1 & 4 & 4 \\ 4 & 0 & 3 \\ 1 & 2 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}.$$ 

We want to find $y \in \mathbb{K}^3$ such that $My = b$, by using Wiedemann’s algorithm.

1. For the choice $u = (1, 0, 0)^T$, show that the algorithm computes the sequence $(3, 0, 4, 2, 3, 0, \ldots)$, then its minimal polynomial $x^2 + 2x + 2$, and that it eventually rejects this choice of $u$.

2. Apply the algorithm for the choice $u = (1, 2, 0)^T$, and deduce that the minimal polynomial of $(M^ib)_{i\geq 0}$ equals $x^3 + 3x + 1$.

3. Determine the solution $y$ by using this minimal polynomial.