Same integrals as in Demo-redct, done with C. Koutschan's FindCreativeTelescoping

```
<< RISC`HolonomicFunctions`
```

HolonomicFunctions Package version 1.7.3 (21-Mar-2017)
written by Christoph Koutschan
Copyright Research Institute for Symbolic Computation (RISC),
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```
--> Type  ?HolonomicFunctions  for help.
```

```
TimeBound = 60 * 60 ;
```

```
Test[f_, var_, rest_] := TimeConstrained[Timing[FindCreativeTelescoping[
    Annihilator[f, Union[{var}, rest]], var, rest]], TimeBound]
```

Examples from C. Koutschan's Examples11.nb

I. Easy ones

```
st = TimeUsed[];
```

```
Test[(LegendreP[2 k + 1, x] / x) ^2, Der[x], {S[k]}]
```

$$\left\{0.605705, \left\{\{S_k - 1\}, \left\{\left\{\frac{-x^2 + x^4}{2\,(3 + 5\,k + 2\,k^2)}\,D_x - \frac{x}{5 + 4\,k}\,S_k + \frac{-4\,x - 3\,k\,x + 5\,x^3 + 4\,k\,x^3}{5 + 9\,k + 4\,k^2}\right\}\right\}\right\}\right\}$$

```
Test[Sqrt[1 - m * u^2] ^ (2 j - 1) / Sqrt[1 - u^2], Der[u], {S[j]}]
```

$$\left\{0.191199, \left\{\left\{(-3 - 2\,j)\,S_j^2 + (4 + 4\,j - 2\,m - 2\,j\,m)\,S_j + (-1 - 2\,j + m + 2\,j\,m)\right\}, \left\{\left\{m\,u - m\,u^3 - m^2\,u^3 + m^2\,u^5\right\}\right\}\right\}\right\}$$

```
Test[ArcCos[x / Sqrt[(a + b) * x - a * b]], Der[x], {Der[a], Der[b]}]
```

$$\left\{0.535907, \left\{\left\{\left(a^2-b^2\right) D_b + (3a+b), \left(-a^2+b^2\right) D_a + (a+3b)\right\}, \left\{\left\{\frac{4\left(a^3 b^2 - 2 a^3 b x - 2 a^2 b^2 x + a^3 x^2 + 3 a^2 b x^2 + a b^2 x^2 - a^2 x^3 - a b x^3\right)}{2 a b - a x - b x} D_x + \left(a^2 + a b - 3 a x - b x\right)\right\}, \left\{\frac{4\left(a^2 b^3 - 2 a^2 b^2 x - 2 a b^3 x + a^2 b x^2 + 3 a b^2 x^2 + b^3 x^2 - a b x^3 - b^2 x^3\right)}{2 a b - a x - b x} D_x + \left(a b + b^2 - a x - 3 b x\right)\right\}\right\}\right\}\right\}$$

```
Test[u^(2 m) / Sqrt[1 - u^2], Der[u], {S[m]}]
```

$$\left\{0.111811, \left\{\left\{(-2-2m) S_m + (1+2m)\right\}, \left\{\left\{-u+u^3\right\}\right\}\right\}\right\}$$

```
Test[1 / (x^4 + 2 a x^2 + 1) ^ (m + 1), Der[x], {S[m], Der[a]}]
```

$$\left\{0.238984, \left\{\left\{(-4-4m) S_m + 2 a D_a + (3+4m), \left(-4+4a^2\right) D_a^2 + (12a + 8am) D_a + (3+4m)\right\}, \left\{\{x\}, \left\{\frac{-3x - 4mx - 2ax^3 - 4amx^3 + x^5}{1 + 2ax^2 + x^4}\right\}\right\}\right\}\right\}$$

```
Test[LegendreP[2 n, u] / Sqrt[1 - u^2], Der[u], {S[n]}]
```

$$\left\{0.285672, \left\{\left\{\left(-4-8n-4n^2\right) S_n + \left(1+4n+4n^2\right)\right\}, \left\{\left\{\frac{2\left(-1-n+u^2+nu^2\right)}{u} S_n + \frac{-1-2n+u^2+2nu^2}{u}\right\}\right\}\right\}\right\}$$

```
Test[Sin[m * x] * Sin[n * x], Der[x], {S[m], S[n]}]
```

Annihilator: The expression Sin[m*x]w.r.t. {S[m]} is not recognized to be ∂–finite. The result might not generate a zero–dimensional ideal.

Annihilator: The expression Sin[n*x]w.r.t. {S[n]} is not recognized to be ∂–finite. The result might not generate a zero–dimensional ideal.

FindCreativeTelescoping: The input does not generate a zero–dimensional ideal.

```
{0.80644, $Failed}
```

```
Test[Exp[-u (a + 1)] * Log[u], Der[a], {Der[u]}]
```

$$\left\{0.076541, \left\{\{1\}, \left\{\left\{\frac{1}{u}\right\}\right\}\right\}\right\}$$

Victor Moll’s integral: special case s=4.

```
ff = (x^2 / (x^4 + 2 a x^2 + 1)) ^ r * (x^2 + 1) / x^2 / (x^s + 1) /. s → 4
```

$$\frac{\left(1+x^2\right)\left(\frac{x^2}{1+2 a x^2+x^4}\right)^r}{x^2\left(1+x^4\right)}$$

```
Test[ff, Der[x], {Der[a], S[r]}]
```

$\{0.57723, \{\{D_a + 2\,r\,S_r,\ (64\,a - 64\,a^3 + 96\,a\,r - 96\,a^3\,r + 32\,a\,r^2 - 32\,a^3\,r^2)\,S_r^3 + (-32 - 16\,a + 64\,a^2 - 48\,r - 16\,a\,r + 112\,a^2\,r - 16\,r^2 + 48\,a^2\,r^2)\,S_r^2 + (8 - 10\,a + 8\,r - 40\,a\,r - 24\,a\,r^2)\,S_r + (-3 + 4\,r + 4\,r^2)\},$
$\{\{0\}, \{(-3\,x - 2\,r\,x - 5\,x^3 - 2\,a\,x^3 - 2\,r\,x^3 - 2\,x^5 - 6\,a\,x^5 - 6\,x^7 - 2\,a\,x^7 + x^9 - 6\,a\,x^9 + 2\,r\,x^9 - x^{11} + 2\,r\,x^{11}) \big/ ((1 + x^2)\,(1 + 2\,a\,x^2 + x^4)^2)\}\}\}\}$

```
Test[ArcSinh[x] / Sqrt[x^2 + 1] * E^(-z * x), Der[x], {Der[z]}]
```

$\{0.246354, \{\{z\,D_z^2 + D_z + z\}, \{\{\frac{1 + x^2}{z}\,D_x + \frac{x + 2\,z + 2\,x^2\,z}{z}\}\}\}\}$

```
Test[z / 2 * ArcSinh[x]^2 * E^(-z * x), Der[x], {Der[z]}]
```

$\{0.342902, \{\{z\,D_z^2 + D_z + z\}, \{\{\frac{1 + x^2}{z^2}\,D_x^2 + \frac{x + 3\,z + 3\,x^2\,z}{z^2}\,D_x + 3\,(1 + x^2)\}\}\}\}$

```
Test[x * ArcSinh[x] / Sqrt[x^2 + 1] * E^(-z * x) / 2, Der[x], {Der[z]}]
```

$\{0.323606, \{\{z^2\,D_z^2 + z\,D_z + (-1 + z^2)\},$
$\{\{\frac{2 + 2\,x^2 + x\,z + x^3\,z}{x\,z}\,D_x + \frac{1}{x^2\,z}(-2 + 2\,x\,z + 3\,x^3\,z + 2\,x^2\,z^2 + 2\,x^4\,z^2)\}\}\}\}$

```
Test[ArcSinh[x] / (1 + x^2)^(n + 1), Der[x], {S[n]}]
```

$\{0.26717, \{\{(-2 - 2\,n)\,S_n + (1 + 2\,n)\}, \{\{\frac{1 + x^2}{1 + 2\,n}\,D_x + \frac{(3 + 4\,n)\,x}{1 + 2\,n}\}\}\}\}$

```
annLommel = ToOrePolynomial[
  {S[n] - 1, Der[x] ** (x^2 * Der[x]^2 + x * Der[x] + x^2)}, OreAlgebra[Der[x], S[n]]]
```

$\{S_n - 1,\ x^2\,D_x^3 + 3\,x\,D_x^2 + (1 + x^2)\,D_x + 2\,x\}$

```
integrand =
 DFiniteTimes[Annihilator[x^n * BesselJ[n, x], {Der[x], S[n]}], annLommel]
```

$\{S_n^2 + (-2 - 2\,n)\,S_n + x^2,\ x^3\,D_x^3 + 3\,x^2\,D_x^2\,S_n + (3\,x^2 - 6\,n\,x^2)\,D_x^2 + (3\,x - 6\,n\,x)\,D_x\,S_n + (x - 6\,n\,x + 12\,n^2\,x - 2\,x^3)\,D_x + 4\,n^2\,S_n + (-8\,n^3 - 2\,x^2 + 2\,n\,x^2)\}$

```
ct = CreativeTelescoping[integrand, Der[x], S[n]]
```

$\{\{(4 + 2\,n)\,S_n^2 + (-11 - 18\,n - 8\,n^2)\,S_n + (1 + 6\,n + 12\,n^2 + 8\,n^3)\},$
$\{-x^3\,D_x^2 - 3\,x^2\,D_x\,S_n + 6\,n\,x^2\,D_x + 3\,(x + 2\,n\,x)\,S_n + (-x - 6\,n\,x - 12\,n^2\,x + 2\,x^3)\}\}$

```
Test[(1 + x t + t^2)^(-2), Der[t], {Der[x]}]
```

$\{0.08994, \{\{(-4 + x^2)\,D_x + 3\,x\}, \{\{2 + t\,x\}\}\}\}$

```
Test[1 / (1 + x^2)^n, Der[x], { S[n]}]
```

$\{0.073431, \{\{-2\,n\,S_n + (-1 + 2\,n)\}, \{\{x\}\}\}\}$

```
Test[x^(m-1)*Exp[-c*x-b*x^2]*Sin[a*x],
 Der[x], {S[m], Der[a], Der[b], Der[c]}]
```

$$\Big\{0.629041,\ \Big\{\Big\{a\,D_a+2\,b\,D_b+c\,D_c+m,\ S_m+D_c,\ D_c^2+D_b,\ 4\,b^2\,D_b^2+4\,b\,c\,D_b\,D_c+\left(-a^2+6\,b-c^2+4\,b\,m\right)D_b+(2\,c+2\,c\,m)\,D_c+\left(m+m^2\right)\Big\},\ \Big\{\{-x\},\{0\},\{0\},\left\{x^2\,D_x-2\left(m\,x-c\,x^2-2\,b\,x^3\right)\right\}\Big\}\Big\}\Big\}$$

```
Test[ArcTan[p*x]/(1+p^2*x), Der[x], {Der[p]}]
```

$$\Big\{0.347458,\ \Big\{\Big\{\left(-p^2-p^4\right)D_p^2+\left(-4\,p-6\,p^3\right)D_p+\left(-2-6\,p^2\right)\Big\},\ \Big\{\Big\{\frac{-1+p^4\,x^2}{p^2}\,D_x+\frac{2\left(-1+x+2\,p^2\,x-p^2\,x^2+p^4\,x^2\right)}{1+p^2\,x}\Big\}\Big\}\Big\}\Big\}$$

```
Test[BesselJ[m, a*x]*BesselJ[n, b*x], Der[x], {S[n], S[m], Der[a], Der[b]}]
```

$$\Big\{1.27481,\ \Big\{\Big\{-a\,D_a-b\,D_b-1,\ \left(a^2\,b^2-b^4\right)D_b^2+\left(a^2\,b-3\,b^3\right)D_b+\left(-b^2+b^2\,m^2-a^2\,n^2\right),\ \left(a^2\,b-b^3\right)S_m\,D_b+(a\,b\,m-a\,b\,n)\,S_n+\left(b^2\,m-a^2\,n\right)S_m,\ \left(-a^2-2\,a^2\,m-a^2\,m^2+a^2\,n^2\right)S_m^2+\left(2\,a^2\,b-2\,b^3+2\,a^2\,b\,m-2\,b^3\,m\right)D_b+\left(a^2-2\,b^2+2\,a^2\,m-4\,b^2\,m+a^2\,m^2-2\,b^2\,m^2+a^2\,n^2\right),\ \left(-a^2\,b+b^3\right)S_n\,D_b+\left(-a^2+b^2+b^2\,m-a^2\,n\right)S_n+(a\,b\,m-a\,b\,n)\,S_m,\ (a\,b+a\,b\,m+a\,b\,n)\,S_n\,S_m+\left(a^2\,b-b^3\right)D_b+\left(-b^2-b^2\,m-a^2\,n\right),\ \left(-b^2+b^2\,m^2-2\,b^2\,n-b^2\,n^2\right)S_n^2+\left(2\,a^2\,b-2\,b^3+2\,a^2\,b\,n-2\,b^3\,n\right)D_b+\left(-b^2+b^2\,m^2-2\,a^2\,n-2\,a^2\,n^2+b^2\,n^2\right)\Big\},\ \Big\{\{x\},\left\{a\,b^2\,x^2\,S_m-b^3\,x^2\,S_n-\left(-b^2+b^2\,m-b^2\,n\right)x\right\},\left\{b^2\,x\,S_m-a\,b\,x\,S_n\right\},\ \left\{2\,(a\,b+a\,b\,m)\,x\,S_m\,S_n-2\left(a+2\,a\,m+a\,m^2+a\,n+a\,m\,n\right)S_m+2\left(b^2+b^2\,m\right)x\right\},\ \left\{a\,b\,x\,S_m-b^2\,x\,S_n\right\},\left\{a\,b\,x\,S_m\,S_n+b^2\,x\right\},\ \left\{2\,(a\,b+a\,b\,n)\,x\,S_m\,S_n-2\left(b+b\,m+2\,b\,n+b\,m\,n+b\,n^2\right)S_n+2\left(b^2+b^2\,n\right)x\right\}\Big\}\Big\}\Big\}$$

```
Test[Log[(Sqrt[x^2+a^2]+x)/(Sqrt[x^2+a^2]-x)]*
  BesselJ[0, b*x]/Sqrt[x^2+a^2], Der[x], {Der[a], Der[b]}]
```

$$\Big\{1.29586,\ \Big\{\Big\{-a\,D_a+b\,D_b,\ b^2\,D_b^3+3\,b\,D_b^2+\left(1-a^2\,b^2\right)D_b-a^2\,b\Big\},\ \Big\{\{-x\},\Big\{\frac{a^2\,x+x^3}{2\,b}\,D_x^2+\frac{a^2}{2\,x}\,D_b+\frac{3\,x^2}{2\,b}\,D_x+\frac{x+3\,a^2\,b^2\,x+3\,b^2\,x^3}{2\,b}\Big\}\Big\}\Big\}\Big\}$$

```
Test[(1-x^2)^(nu-1/2)*GegenbauerC[m, nu, x]*GegenbauerC[n, nu, x],
 Der[x], {S[n], S[m], S[nu]}]
```

$$\Big\{0.567152,\ \Big\{\{1\},\ \Big\{\Big\{\frac{-1-m}{(m-n)\,(m+n+2\,nu)}\,S_m+\frac{-1-n}{(-m+n)\,(m+n+2\,nu)}\,S_n+\frac{x}{m+n+2\,nu}\Big\}\Big\}\Big\}\Big\}$$

```
Test[(x (2 a - x)) ^ (nu - 1 / 2) * GegenbauerC[n, nu, x / a - 1] * E ^ (-b * x),
 Der[x], {Der[a], Der[b], S[n], S[nu]}]
```

$$\Big\{0.920638, \Big\{\Big\{\left(a + 2\,a\,n + a\,n^2 + 2\,a\,nu + 2\,a\,n\,nu\right) S_n + 2\,b\,nu\,S_{nu},$$
$$\left(b\,n + b\,n^2 + 2\,b\,nu + 4\,b\,n\,nu + 4\,b\,nu^2\right) D_b - 2\,b^2\,nu\,S_{nu} +$$
$$\left(a\,b\,n - n^2 + a\,b\,n^2 - n^3 + 2\,a\,b\,nu - 2\,n\,nu + 4\,a\,b\,n\,nu - 4\,n^2\,nu + 4\,a\,b\,nu^2 - 4\,n\,nu^2\right),$$
$$\left(a\,n + a\,n^2 + 2\,a\,nu + 4\,a\,n\,nu + 4\,a\,nu^2\right) D_a - 2\,b^2\,nu\,S_{nu} + \big(a\,b\,n - n^2 + a\,b\,n^2 - n^3 +$$
$$2\,a\,b\,nu - 4\,n\,nu + 4\,a\,b\,n\,nu - 6\,n^2\,nu - 4\,nu^2 + 4\,a\,b\,nu^2 - 12\,n\,nu^2 - 8\,nu^3\big),$$
$$\left(4\,b^2\,nu + 4\,b^2\,nu^2\right) S_{nu}^2 + \big(24\,nu + 44\,n\,nu + 24\,n^2\,nu + 4\,n^3\,nu + 64\,nu^2 +$$
$$76\,n\,nu^2 + 20\,n^2\,nu^2 + 56\,nu^3 + 32\,n\,nu^3 + 16\,nu^4\big) S_{nu} +$$
$$\big(-6\,a^2\,n - 11\,a^2\,n^2 - 6\,a^2\,n^3 - a^2\,n^4 - 12\,a^2\,nu - 44\,a^2\,n\,nu - 36\,a^2\,n^2\,nu - 8\,a^2\,n^3\,nu -$$
$$44\,a^2\,nu^2 - 72\,a^2\,n\,nu^2 - 24\,a^2\,n^2\,nu^2 - 48\,a^2\,nu^3 - 32\,a^2\,n\,nu^3 - 16\,a^2\,nu^4\big)\Big\},$$
$$\Big\{\{2\,nu\,S_{nu}\}, \Big\{-\frac{1}{a-x} 2\left(-nu + a\,b\,nu - n\,nu - 2\,nu^2 - b\,nu\,x\right) S_{nu} + \frac{1}{a-x}\big(-2\,a\,n\,x -$$
$$2\,a\,n^2\,x - 4\,a\,nu\,x - 8\,a\,n\,nu\,x - 8\,a\,nu^2\,x + n\,x^2 + n^2\,x^2 + 2\,nu\,x^2 + 4\,n\,nu\,x^2 + 4\,nu^2\,x^2\big)\Big\},$$
$$\Big\{-\frac{1}{a-x} 2\left(-nu + a\,b\,nu - n\,nu - 2\,nu^2 - b\,nu\,x\right) S_{nu} - \frac{1}{a-x}$$
$$\left(a\,n + a\,n^2 + 2\,a\,nu + 4\,a\,n\,nu + 4\,a\,nu^2\right) x\Big\},$$
$$\Big\{\frac{1}{a-x} 2\,\big(3\,a^2\,nu + a^3\,b\,nu + 4\,a^2\,n\,nu + a^2\,n^2\,nu + 2\,a^2\,nu^2 + 2\,a^3\,b\,nu^2 + 2\,a^2\,n\,nu^2 - 12\,a\,nu\,x +$$
$$3\,a^2\,b\,nu\,x - 16\,a\,n\,nu\,x + 4\,a^2\,b\,n\,nu\,x - 4\,a\,n^2\,nu\,x - 20\,a\,nu^2\,x + 2\,a^2\,b\,nu^2\,x -$$
$$12\,a\,n\,nu^2\,x - 8\,a\,nu^3\,x + 6\,nu\,x^2 - 6\,a\,b\,nu\,x^2 + 8\,n\,nu\,x^2 - 6\,a\,b\,n\,nu\,x^2 + 2\,n^2\,nu\,x^2 +$$
$$10\,nu^2\,x^2 - 6\,a\,b\,nu^2\,x^2 + 6\,n\,nu^2\,x^2 + 4\,nu^3\,x^2 + 2\,b\,nu\,x^3 + 2\,b\,n\,nu\,x^3 + 2\,b\,nu^2\,x^3\big)$$
$$S_{nu} + \frac{1}{a-x}\big(6\,a^3\,n\,x - 2\,a^4\,b\,n\,x + 8\,a^3\,n^2\,x - 2\,a^4\,b\,n^2\,x + 2\,a^3\,n^3\,x + 12\,a^3\,nu\,x -$$
$$4\,a^4\,b\,nu\,x + 32\,a^3\,n\,nu\,x - 8\,a^4\,b\,n\,nu\,x + 12\,a^3\,n^2\,nu\,x + 32\,a^3\,nu^2\,x -$$
$$8\,a^4\,b\,nu^2\,x + 24\,a^3\,n\,nu^2\,x + 16\,a^3\,nu^3\,x - 3\,a^2\,n\,x^2 + 3\,a^3\,b\,n\,x^2 - 4\,a^2\,n^2\,x^2 +$$
$$3\,a^3\,b\,n^2\,x^2 - a^2\,n^3\,x^2 - 6\,a^2\,nu\,x^2 + 6\,a^3\,b\,nu\,x^2 - 16\,a^2\,n\,nu\,x^2 + 12\,a^3\,b\,n\,nu\,x^2 -$$
$$6\,a^2\,n^2\,nu\,x^2 - 16\,a^2\,nu^2\,x^2 + 12\,a^3\,b\,nu^2\,x^2 - 12\,a^2\,n\,nu^2\,x^2 - 8\,a^2\,nu^3\,x^2 -$$
$$a^2\,b\,n\,x^3 - a^2\,b\,n^2\,x^3 - 2\,a^2\,b\,nu\,x^3 - 4\,a^2\,b\,n\,nu\,x^3 - 4\,a^2\,b\,nu^2\,x^3\big)\Big\}\Big\}\Big\}\Big\}$$

```
Test[ChebyshevT[n, 1 - x^2 y] / Sqrt[1 - x^2], Der[x], {S[n], Der[y]}]
```

$$\Big\{0.702383, \Big\{\Big\{\left(-2\,n - 2\,n^2\right) S_n + \left(2\,y + 4\,n\,y - y^2 - 2\,n\,y^2\right) D_y + \left(2\,n + 2\,n^2 - n\,y - 2\,n^2\,y\right),$$
$$\left(2\,y - y^2\right) D_y^2 + \left(2 - y\right) D_y + n^2\Big\},$$
$$\Big\{\Big\{\frac{-n + n\,x^2}{x} S_n + \frac{n - n\,x^2}{x}\Big\}, \Big\{\frac{-n + n\,x^2}{x\,y\left(-2 + x^2\,y\right)} S_n + \frac{n - n\,x^2 - n\,x^2\,y + n\,x^4\,y}{x\,y\left(-2 + x^2\,y\right)}\Big\}\Big\}\Big\}\Big\}$$

```
Test[x^(r-1) * (1-x)^(s-1) * Hypergeometric2F1[a, b, c, x],
 Der[x], {S[a], S[b], S[c], S[r], S[s]}]
```

$$\Big\{1.01152, \Big\{\Big\{S_r + S_s - 1,\ (a\,b\,c - a\,c^2 - b\,c^2 + c^3 - a\,b\,r + a\,c\,r + b\,c\,r - c^2\,r)\,S_c + (-a\,b\,c + a\,c\,r + b\,c\,r - c\,r^2 + a\,c\,s + b\,c\,s - 2\,c\,r\,s - c\,s^2)\,S_s + (a\,c^2 + b\,c^2 - c^3 - a\,c\,r - b\,c\,r + c^2\,r - a\,c\,s - b\,c\,s + c\,r\,s + c\,s^2),$$

$$(b + a\,b + b^2 - b\,c - b\,s)\,S_b + (-a\,b + a\,r + b\,r - r^2 + a\,s + b\,s - 2\,r\,s - s^2)\,S_s + (-b - b^2 + b\,c - a\,s + r\,s + s^2),\ (a + a^2 + a\,b - a\,c - a\,s)\,S_a + (-a\,b + a\,r + b\,r - r^2 + a\,s + b\,s - 2\,r\,s - s^2)\,S_s + (-a - a^2 + a\,c - b\,s + r\,s + s^2),$$

$$(1 - a - b + a\,b + 2\,r - a\,r - b\,r + r^2 + 2\,s - a\,s - b\,s + 2\,r\,s + s^2)\,S_s^2 + (-1 + a + b - a\,b - r + a\,r + b\,r - c\,r - 2\,s + 2\,a\,s + 2\,b\,s - c\,s - 2\,r\,s - 2\,s^2)\,S_s + (-a\,s - b\,s + c\,s + s^2)\Big\},$$

$$\Big\{\{0\}, \Big\{(a\,b\,x - a\,c\,x - b\,c\,x + c^2\,x - a\,b\,x^2 + a\,c\,x^2 + b\,c\,x^2 - c^2\,x^2)\,S_c + (-c^2\,x + c\,r\,x + c\,s\,x + c^2\,x^2 - c\,r\,x^2 - c\,s\,x^2)\Big\},$$

$$\Big\{-\frac{(a\,b - a\,c - b\,c + c^2)\,x^2}{c}\,S_c + (-a\,x - b\,x + r\,x + s\,x + c\,x^2 - r\,x^2 - s\,x^2)\Big\},$$

$$\Big\{-\frac{(a\,b - a\,c - b\,c + c^2)\,x^2}{c}\,S_c + (-a\,x - b\,x + r\,x + s\,x + c\,x^2 - r\,x^2 - s\,x^2)\Big\},$$

$$\Big\{\frac{1}{c}(a\,b\,x^2 - a\,c\,x^2 - b\,c\,x^2 + c^2\,x^2 - a\,b\,x^3 + a\,c\,x^3 + b\,c\,x^3 - c^2\,x^3)\,S_c + (-x + c\,x - r\,x + 2\,x^2 - 2\,c\,x^2 + 2\,r\,x^2 + s\,x^2 - x^3 + c\,x^3 - r\,x^3 - s\,x^3)\Big\}\Big\}\Big\}\Big\}$$

```
Test[(z/2)^n / Gamma[n+1/2] / Gamma[1/2] * (1-t^2)^(n-1/2) * Cos[z t],
 Der[t], {Der[z], S[n]}]
```

$$\Big\{0.484635, \Big\{\{-z\,D_z - z\,S_n + n,\ z\,S_n^2 + (-2 - 2\,n)\,S_n + z\},\ \Big\{\Big\{\frac{-z + t^2\,z}{(1 + 2\,n)\,t}\,D_z + \frac{n - n\,t^2}{(1 + 2\,n)\,t}\Big\},$$

$$\Big\{\frac{z^2 - 2\,t^2\,z^2 + t^4\,z^2}{(3 + 8\,n + 4\,n^2)\,t}\,D_z + (-n\,z + 3\,t^2\,z + 4\,n\,t^2\,z - 3\,t^4\,z - 3\,n\,t^4\,z) \big/ \big((3 + 8\,n + 4\,n^2)\,t\big)\Big\}\Big\}\Big\}\Big\}$$

```
Test[t^(-n-1) * Exp[t - z^2/(4 t)], Der[t], {S[n], Der[z]}]
```

$$\{0.154007, \{\{z\,S_n + 2\,D_z,\ -z\,D_z^2 + (-1 - 2\,n)\,D_z - z\},\ \{\{0\},\ \{z\}\}\}\}$$

```
Test[Sin[z t] (1-t^2)^(n+1/2), Der[t], {S[n], Der[z]}]
```

$$\Big\{0.395947, \Big\{\{z\,S_n + (3 + 2\,n)\,D_z,\ -z\,D_z^2 + (-3 - 2\,n)\,D_z - z\},\ \Big\{\Big\{\frac{1 - t^2}{t}\,D_z\Big\},\ \Big\{\frac{-1 + t^2}{t}\,D_z\Big\}\Big\}\Big\}\Big\}$$

```
Test[Exp[-z t] (1+t^2)^(n-1/2), Der[t], {S[n], Der[z]}]
```

$$\{0.173728, \{\{z\,S_n + (1 + 2\,n)\,D_z,\ z\,D_z^2 + (1 + 2\,n)\,D_z + z\},\ \{\{1 + t^2\},\ \{1 + t^2\}\}\}\}$$

```
Test[BesselJ[n, b t] * Exp[-p^2 * t^2] * t^(n+1), Der[t], {Der[b], S[n], Der[p]}]
```

$$\Big\{0.335632, \Big\{\{2\,p^3\,D_p + (-b^2 + 4\,p^2 + 4\,n\,p^2),\ 2\,p^2\,S_n - b,\ -2\,b\,p^2\,D_b + (-b^2 + 2\,n\,p^2)\},$$

$$\Big\{\Big\{\frac{b}{t}\,S_n - 2\,p^2\,t\Big\},\ \Big\{\frac{1}{t}\,S_n\Big\},\ \Big\{\frac{b}{t}\,S_n\Big\}\Big\}\Big\}\Big\}$$

```
Test[t^n * BesselY[n, a t] / (t^2 + k^2), Der[t], {S[n], Der[a], Der[k]}]
```

$$\left\{0.690229, \left\{\left\{-a D_a + k D_k + (1-n),\ a S_n + k D_k + (1-2n),\ -k^2 D_k^3 + (-5k + 2kn) D_k^2 + (-4 + a^2 k^2 + 4n) D_k + 2a^2 k\right\}, \left\{\{t\}, \{t\}, \left\{-\frac{2akt}{k^2+t^2} S_n - \frac{2(k^3 t - 3kt^3)}{(k^2+t^2)^2}\right\}\right\}\right\}\right\}$$

```
Test[Exp[-p^2 * t^2] * BesselJ[0, a t] * BesselY[0, a t] * t, Der[t], {Der[a], Der[p]}]
```

$$\left\{0.597023, \left\{\left\{a D_a + p D_p + 2,\ p^4 D_p^2 + (-2a^2 p + 5p^3) D_p + (-2a^2 + 4p^2)\right\}, \left\{\{-t\}, \left\{\frac{t}{2} D_t^2 + \frac{1}{2}(-1 + 6p^2 t^2) D_t + \frac{1 + 4a^2 t^2 - 6p^2 t^2 + 12 p^4 t^4}{2t}\right\}\right\}\right\}\right\}$$

```
Test[z^(n + 1) * BesselI[n, z], Der[z], {S[n]}]
```

$$\left\{0.110406, \left\{\{1\}, \left\{\left\{-\frac{1}{z} S_n\right\}\right\}\right\}\right\}$$

```
Test[t^(a - 1) * (1 - t)^(b - 1), Der[t], {S[a], S[b]}]
```

$$\left\{0.135683, \left\{\{(a+b) S_b - b,\ (a+b) S_a - a\}, \left\{\{-t + t^2\}, \{t - t^2\}\right\}\right\}\right\}$$

```
Test[Exp[-a t] * StruveH[0, t], Der[t], {Der[a]}]
```

$$\left\{0.199298, \left\{\left\{(-1 - a^2) D_a - a\right\}, \left\{\left\{\frac{t}{a} D_t^2 + \frac{1 + 3at}{a} D_t + \frac{a + t + 3a^2 t}{a}\right\}\right\}\right\}\right\}$$

```
Test[AiryAi[2^(2 / 3) * (u^2 + x)], Der[u], {Der[x]}]
```

$$\left\{0.132019, \left\{\left\{-D_x^3 + 4x D_x + 2\right\}, \{\{2u\}\}\right\}\right\}$$

```
TimeUsed[] - st
```

15.9946

2. A longer one

```
Test[2 / Pi * BesselJ[m + n, 2 z t] * ChebyshevT[m - n, t] / Sqrt[1 - t^2],
 Der[t], {S[m], S[n], Der[z]}]
```

$$\left\{1.9194, \left\{\left\{z S_m + z S_n + z D_z + (-m-n),\ 2z^2 S_n D_z + z^2 D_z^2 + 2z S_n + (z - 2nz) D_z + (-m^2 + n^2),\ -z S_n^2 + (2 + 2n) S_n - z,\ z^3 D_z^3 + 3z^2 D_z^2 + (2m^2 z - 2n^2 z) S_n + (z - m^2 z - 3n^2 z + 4z^3) D_z + (-2m^2 n + 2n^3 + 4z^2)\right\}, \left\{\{0\}, \{tz S_m - tz S_n\}, \left\{\frac{1}{2t} S_m + \frac{1 - 2t^2}{2t} S_n\right\}, \left\{-t(mz - nz) S_m + t(mz - nz) S_n + 4(-tz^2 + t^3 z^2)\right\}\right\}\right\}\right\}$$

3. Examples that are hard for redct, or where it needs help

```
Test[GegenbauerC[l, λ, x] * GegenbauerC[m, λ, x] *
  GegenbauerC[n, λ, x] * (1 - x^2)^(λ - 1 / 2), Der[x], {S[m], S[n], S[l]}]
```

$$\left\{2.35558, \left\{\left\{(-1 + 2l - l^2 + m^2 - 2n + 2ln - n^2 + 2\lambda - 2l\lambda + 2m\lambda + 2n\lambda) S_n + (1 + 2l + l^2 - m^2 - 2n - 2ln + n^2 - 2\lambda - 2l\lambda - 2m\lambda + 2n\lambda) S_l,\right.\right.\right.$$

$$\left(-1+2 l-l^2-2 m+2 l m-m^2+n^2+2 \lambda-2 l \lambda+2 m \lambda+2 n \lambda\right) S_m+$$
$$\left(1+2 l+l^2-2 m-2 l m+m^2-n^2-2 \lambda-2 l \lambda+2 m \lambda-2 n \lambda\right) S_l,$$
$$\left(16+32 l+24 l^2+8 l^3+l^4-8 m^2-8 l m^2-2 l^2 m^2+m^4-8 n^2-8 l n^2-2 l^2 n^2-\right.$$
$$2 m^2 n^2+n^4-16 m \lambda-16 l m \lambda-4 l^2 m \lambda+4 m^3 \lambda-16 n \lambda-16 l n \lambda-4 l^2 n \lambda-$$
$$\left.4 m^2 n \lambda-4 m n^2 \lambda+4 n^3 \lambda-16 \lambda^2-16 l \lambda^2-4 l^2 \lambda^2+4 m^2 \lambda^2-8 m n \lambda^2+4 n^2 \lambda^2\right) S_l^2+$$
$$\left(-l^4+2 l^2 m^2-m^4+2 l^2 n^2+2 m^2 n^2-n^4-8 l^3 \lambda+4 l^2 m \lambda+8 l m^2 \lambda-4 m^3 \lambda+\right.$$
$$4 l^2 n \lambda+4 m^2 n \lambda+8 l n^2 \lambda+4 m n^2 \lambda-4 n^3 \lambda-20 l^2 \lambda^2+16 l m \lambda^2+$$
$$\left.4 m^2 \lambda^2+16 l n \lambda^2+8 m n \lambda^2+4 n^2 \lambda^2-16 l \lambda^3+16 m \lambda^3+16 n \lambda^3\right)\Big\},$$
$$\Big\{\{(1+m) S_l S_m+(l-n) S_l S_n+(-1-m) S_m S_n-(1+l+m-n) x S_l-$$
$$(-1+l-m-n) x S_n+(l-n)\},\ \{(l-m) S_l S_m+(1+n) S_l S_n+$$
$$(-1-n) S_m S_n-(1+l-m+n) x S_l-(-1+l-m-n) x S_m+(l-m)\},$$
$$\Big\{-4 (1+l+m+l m+n+l n+m n+l m n+\lambda+m \lambda+n \lambda+m n \lambda) S_l S_m S_n+$$
$$\frac{1}{2+l} 2 x \left(4+8 l+5 l^2+l^3+4 m+8 l m+5 l^2 m+l^3 m-m^2-l m^2-m^3-l m^3+4 n+6 l n+\right.$$
$$2 l^2 n+4 m n+6 l m n+2 l^2 m n+n^2+l n^2+m n^2+l m n^2+8 \lambda+10 l \lambda+3 l^2 \lambda+$$
$$6 m \lambda+8 l m \lambda+3 l^2 m \lambda-3 m^2 \lambda-2 l m^2 \lambda-m^3 \lambda+6 n \lambda+4 l n \lambda+6 m n \lambda+4 l m n \lambda+$$
$$\left.n^2 \lambda+m n^2 \lambda+4 \lambda^2+2 l \lambda^2+2 m \lambda^2+2 l m \lambda^2-2 m^2 \lambda^2+2 n \lambda^2+2 m n \lambda^2\right) S_l S_m+$$
$$\frac{1}{2+l} 2 x \left(4+8 l+5 l^2+l^3+4 m+6 l m+2 l^2 m+m^2+l m^2+4 n+8 l n+5 l^2 n+l^3 n+\right.$$
$$4 m n+6 l m n+2 l^2 m n+m^2 n+l m^2 n-n^2-l n^2-n^3-l n^3+8 \lambda+10 l \lambda+3 l^2 \lambda+$$
$$6 m \lambda+4 l m \lambda+m^2 \lambda+6 n \lambda+8 l n \lambda+3 l^2 n \lambda+6 m n \lambda+4 l m n \lambda+m^2 n \lambda-3 n^2 \lambda-$$
$$\left.2 l n^2 \lambda-n^3 \lambda+4 \lambda^2+2 l \lambda^2+2 m \lambda^2+2 n \lambda^2+2 l n \lambda^2+2 m n \lambda^2-2 n^2 \lambda^2\right) S_l S_n-$$
$$\frac{1}{2+l} 2 \left(-4-10 l-10 l^2-5 l^3-l^4+2 m^2+3 l m^2+l^2 m^2+2 n^2+3 l n^2+l^2 n^2+8 x^2+\right.$$
$$20 l x^2+18 l^2 x^2+7 l^3 x^2+l^4 x^2+4 m x^2+8 l m x^2+5 l^2 m x^2+l^3 m x^2-2 m^2 x^2-$$
$$3 l m^2 x^2-l^2 m^2 x^2-m^3 x^2-l m^3 x^2+4 n x^2+8 l n x^2+5 l^2 n x^2+l^3 n x^2+4 m n x^2+$$
$$6 l m n x^2+2 l^2 m n x^2+m^2 n x^2+l m^2 n x^2-2 n^2 x^2-3 l n^2 x^2-l^2 n^2 x^2+m n^2 x^2+$$
$$l m n^2 x^2-n^3 x^2-l n^3 x^2-4 \lambda-10 l \lambda-10 l^2 \lambda-3 l^3 \lambda+4 m \lambda+6 l m \lambda+2 l^2 m \lambda+$$
$$2 m^2 \lambda+l m^2 \lambda+4 n \lambda+6 l n \lambda+2 l^2 n \lambda+2 n^2 \lambda+l n^2 \lambda+16 x^2 \lambda+28 l x^2 \lambda+$$
$$16 l^2 x^2 \lambda+3 l^3 x^2 \lambda+4 m x^2 \lambda+4 l m x^2 \lambda+l^2 m x^2 \lambda-4 m^2 x^2 \lambda-3 l m^2 x^2 \lambda-$$
$$m^3 x^2 \lambda+4 n x^2 \lambda+4 l n x^2 \lambda+l^2 n x^2 \lambda+8 m n x^2 \lambda+6 l m n x^2 \lambda+m^2 n x^2 \lambda-$$
$$4 n^2 x^2 \lambda-3 l n^2 x^2 \lambda+m n^2 x^2 \lambda-n^3 x^2 \lambda-4 l \lambda^2-2 l^2 \lambda^2+4 m \lambda^2+2 l m \lambda^2+4 n \lambda^2+$$
$$\left.2 l n \lambda^2+8 x^2 \lambda^2+8 l x^2 \lambda^2+2 l^2 x^2 \lambda^2-2 m^2 x^2 \lambda^2+4 m n x^2 \lambda^2-2 n^2 x^2 \lambda^2\right) S_l-$$
$$\frac{1}{2+l} 2 \left(2 l+3 l^2+l^3+2 l m+3 l^2 m+l^3 m-m^2-l m^2-m^3-l m^3+n^2+l n^2+\right.$$
$$m n^2+l m n^2+4 \lambda+8 l \lambda+3 l^2 \lambda+2 m \lambda+6 l m \lambda+3 l^2 m \lambda-3 m^2 \lambda-$$
$$2 l m^2 \lambda-m^3 \lambda+2 n \lambda+2 l n \lambda+2 m n \lambda+2 l m n \lambda+n^2 \lambda+m n^2 \lambda+$$
$$\left.4 \lambda^2+2 l \lambda^2+2 m \lambda^2+2 l m \lambda^2-2 m^2 \lambda^2+2 n \lambda^2+2 m n \lambda^2\right) S_m-$$
$$\frac{1}{2+l} 2 \left(2 l+3 l^2+l^3+m^2+l m^2+2 l n+3 l^2 n+l^3 n+m^2 n+l m^2 n-n^2-\right.$$
$$l n^2-n^3-l n^3+4 \lambda+8 l \lambda+3 l^2 \lambda+2 m \lambda+2 l m \lambda+m^2 \lambda+2 n \lambda+$$
$$6 l n \lambda+3 l^2 n \lambda+2 m n \lambda+2 l m n \lambda+m^2 n \lambda-3 n^2 \lambda-2 l n^2 \lambda-$$
$$\left.n^3 \lambda+4 \lambda^2+2 l \lambda^2+2 m \lambda^2+2 n \lambda^2+2 l n \lambda^2+2 m n \lambda^2-2 n^2 \lambda^2\right) S_n+$$
$$\frac{1}{2+l} 2 x \left(4 l+6 l^2+2 l^3+2 l m+3 l^2 m+l^3 m-m^3-l m^3+2 l n+3 l^2 n+\right.$$

$l^3 n + m^2 n + l m^2 n + m n^2 + l m n^2 - n^3 - l n^3 + 8 \lambda + 16 l \lambda + 6 l^2 \lambda + 4 m \lambda +$
$8 l m \lambda + 3 l^2 m \lambda - 2 m^2 \lambda - 2 l m^2 \lambda - m^3 \lambda + 4 n \lambda + 8 l n \lambda + 3 l^2 n \lambda +$
$4 m n \lambda + 4 l m n \lambda + m^2 n \lambda - 2 n^2 \lambda - 2 l n^2 \lambda + m n^2 \lambda - n^3 \lambda + 8 \lambda^2 + 4 l \lambda^2 +$
$4 m \lambda^2 + 2 l m \lambda^2 - 2 m^2 \lambda^2 + 4 n \lambda^2 + 2 l n \lambda^2 + 4 m n \lambda^2 - 2 n^2 \lambda^2) \} \} \} \}$

```
Test[x BesselJ[1, a x] BesselI[1, a x] BesselY[0, x] BesselK[0, x], Der[x], {Der[a]}]
```

$$\Big\{5.26713, \{\{-a D_a - 2\}, \{\{\frac{5 a^2}{4(-1+a^4) x} D_a^2 D_x^2 - \frac{5 a}{4(-1+a^4)} D_a D_x^3 + \frac{3 x}{8(-1+a^4)} D_x^4 - \frac{a^3}{(-1+a^4) x^3} D_a^3 - \frac{a^2}{(-1+a^4) x^2} D_a^2 D_x + \frac{3 a}{2(-1+a^4) x} D_a D_x^2 + \frac{1}{2(-1+a^4)} D_x^3 - \frac{5 a^2}{2(-1+a^4) x^3} D_a^2 - \frac{7 a}{2(-1+a^4) x^2} D_a D_x + \frac{3}{8(-1+a^4) x} D_x^2 + \frac{5 a}{(-1+a^4) x^3} D_a - \frac{5}{8(-1+a^4) x^2} D_x + \frac{5 + 12 x^4 + 20 a^4 x^4}{8(-1+a^4) x^3}\}\}\}\Big\}$$

```
Test[x BesselJ[1, a x] BesselI[1, a x] BesselY[0, x] BesselK[0, x], Der[a], {Der[x]}]
```

$$\Big\{3.34815, \{\{x^3 D_x^4 + 4 x^2 D_x^3 + x D_x^2 - D_x + 4 x^3\}, \{\{\frac{a^4}{x} D_a^3 - 4 a^3 D_a^2 D_x + 6 a^2 x D_a D_x^2 - 4 a x^2 D_x^3 + \frac{8 a^3}{x} D_a^2 - 12 a^2 D_a D_x + \frac{13 a^2}{x} D_a - 2 a D_x + \frac{3 a}{x}\}\}\}\Big\}$$

```
Test[(c + I * u * Sqrt[1 - c^2]) ^n / Sqrt[1 - u^2], Der[u], {S[n], Der[c]}]
```

$$\Big\{0.697364, \{\{(1+n) S_n + (1-c^2) D_c + (-c - c n), (-1+c^2) D_c^2 + 2 c D_c + (-n - n^2)\}, \{\{\frac{1-u^2}{u} S_n + \frac{-c + c u^2}{u}\}, \{\frac{-c n + c n u^2}{(-1+c^2) u (-c^2 - u^2 + c^2 u^2)} S_n + \frac{c^2 n - n u^2 + n u^4 - c^2 n u^4}{(-1+c^2) u (-c^2 - u^2 + c^2 u^2)}\}\}\}\Big\}$$

```
Test[Exp[x] * x^(-a / 2) / n! * Exp[-t] * t^(n + a / 2) * BesselJ[a, 2 (t * x) ^ (1 / 2)],
 Der[t], {S[n], S[a], Der[x]}]
```

$$\Big\{0.88894, \{\{(1+n) S_n - x D_x + (-1 - a - n + x), -x D_x^2 + (-1 - a + x) D_x - n, -x S_a^2 + (-1 - a - x) D_x + (-n + x)\}, \{\{t\}, \{t\}, \{t\}\}\}\Big\}$$

Bivariate rational integrands

```
f = 1 / (1 - y - x / y - x (1 - y^2))
```

$$\frac{1}{1 - \frac{x}{y} - y - x (1 - y^2)}$$

Our routine Test does not work directly on these examples, but Christoph Koutschan suggested changing the order of the derivations below, which solves the problem:

```
ann = Annihilator[f, {Der[y], Der[x]}]
```

$\{(-x+y-x\,y-y^2+x\,y^3)\,D_x+(-1-y+y^3),\ (-x\,y+y^2-x\,y^2-y^3+x\,y^4)\,D_y+(x-y^2+2\,x\,y^3)\}$

```
CreativeTelescoping[ann, Der[y], {Der[x]}]
```

$\{\{(-1+15\,x-96\,x^2+335\,x^3-592\,x^4+335\,x^5+345\,x^6)\,D_x^2+(10-88\,x+370\,x^2-852\,x^3+550\,x^4+1380\,x^5)\,D_x+(-12+30\,x+72\,x^2-120\,x^3+690\,x^4)\},$
$\{(x^2-6\,x^3+48\,x^4-155\,x^5-2\,x\,y+14\,x^2\,y-108\,x^3\,y+406\,x^4\,y-310\,x^5\,y+y^2-7\,x\,y^2+59\,x^2\,y^2-219\,x^3\,y^2+213\,x^4\,y^2-285\,x^5\,y^2-2\,y^3+16\,x\,y^3-118\,x^2\,y^3+444\,x^3\,y^3-518\,x^4\,y^3+400\,x^5\,y^3+y^4-5\,x\,y^4+26\,x^2\,y^4-10\,x^3\,y^4-368\,x^4\,y^4+395\,x^5\,y^4-2\,x\,y^5+20\,x^2\,y^5-116\,x^3\,y^5+330\,x^4\,y^5-260\,x^5\,y^5)\,/\,(x^2\,y\,(-x+y-x\,y-y^2+x\,y^3))\}\}$

```
Do[f = 1/(1-y-x/y-x*(1-y^d));
 ann = Annihilator[f, {Der[y], Der[x]}];
 tt = Timing[CreativeTelescoping[ann, Der[y], {Der[x]}]];
 Print[d, "  ", tt[[1]]], {d, 12}]
```

```
1  0.033569
2  0.068004
3  0.209392
4  0.522707
5  1.15317
6  2.32629
7  4.42407
8  7.49206
9  12.6601
10  20.6163
11  32.7544
12  51.1272
```

Bivariate Hyperexponential Integrals

```
f = (62*x-85*y-26)*Sqrt[(-99*x+33*y+62)/(-66*x-63*y+29)]*
  Exp[(-51*x+86*y-54)/(-51*x-12*y+68)]/(-44*x-y+87)^5
```

$$\left(e^{\frac{-54-51\,x+86\,y}{68-51\,x-12\,y}}\,(-26+62\,x-85\,y)\,\sqrt{\frac{62-99\,x+33\,y}{29-66\,x-63\,y}}\right)\Big/\,(87-44\,x-y)^5$$

Here, exchanging the order of the derivations does not help, so we use FindCreativeTelescoping instead of CreativeTelescoping:

```
Timing[FindCreativeTelescoping[f, Der[y], {Der[x]}]][[1]]
```

```
18.5605
```

```
? FindCreativeTelescoping
```

FindCreativeTelescoping[f, {d1, ..., dj}, {op1, ..., opk}] or FindCreativeTelescoping[ann, {d1, ..., dj}] finds creative telescoping relations for the given function f (resp. in the given ∂–finite ideal ann annihilating some function f). In particular it returns {{p1, ..., pm}, {{q11, ..., q1j}, ..., {qm1, ..., qmj}}}, wherein all entries are OrePolynomials such that pi + d1*qi1 + ... + dj*qij is in the annihilator of f for all 1 <= i <= m. FindCreativeTelescoping makes an ansatz with explicit denominators, in contrast to Chyzak's algorithm (see the command CreativeTelescoping) where the denominators have to be computed later. Modular computations are used to get the denominators right.
The following options can be given:
Degree –> n: to limit the degrees of the summation/integration variables in the numerators by n (each variable separately).
Support –> {...}: to give the support of the telescoper.
Mode –> Automatic: finds the right ansatz and does the non–modular computation automatically.
Mode –> FindSupport: does only the modular computations and returns a list of options specifying the ansatz which can again be given to FindCreativeTelescoping again.
Mode –> Modular: takes a fixed ansatz and computes homomorphic images (to be specified by the options OrePolynomialSubstitute and Modulus, and FileNames has to give a StringForm where on the disk to store the results).
Denominator –> d: to give a common denominator that is tried for finding a creative telescoping relation.
"MinimizeDenominators" –> True: whether to minimize the denominators in the delta parts using homomorphic images.

A bigger example:

```
f = (62 * x - 85 * y - 26) * Sqrt[(-99 * x + 33 * y + 62) / (-66 * x - 63 * y + 29)] *
  Exp[(-99 * x^2 + 62 * x * y + 23 * y^2 + 40 * x + 41 * y - 87) /
     (-53 * x^2 + 9 * x * y + 44 * y^2 - 89 * x + 40 * y + 98)] / (-44 * x - y + 87)^5
```

$$\left(e^{\frac{-87+40x-99x^2+41y+62xy+23y^2}{98-89x-53x^2+40y+9xy+44y^2}}\,(-26+62x-85y)\sqrt{\frac{62-99x+33y}{29-66x-63y}}\right)\Big/\,(87-44x-y)^5$$

```
TimeConstrained[
 Timing[FindCreativeTelescoping[f, Der[y], {Der[x]}]], TimeBound]
```

$$\left\{2255.42,\ \left\{\left\{(\cdots 1 \cdots)\, D_x^5 + (\cdots 1 \cdots)\, D_x^4 + (\cdots 1 \cdots)\, D_x^3 + (\cdots 1 \cdots)\, D_x^2 + (\cdots 1 \cdots)\, D_x + (\cdots 1 \cdots)\right\},\ \left\{\left\{\frac{\cdots 1 \cdots}{\cdots 1 \cdots}\right\}\right\}\right\}\right\}$$

large output | show less | show more | show all | set size limit...

Bivariate Mixed Terms

```
f = (1 + x / (n^2 + 1)) * ((x + 1) ^2 / (x - 4) / (x - 3) ^2 / (x^2 - 5) ^3) ^n *
   Sqrt[x^2 - 5] * Exp[(x^3 + 1) / x / (x - 3) / (x - 4) ^2]
```

$$e^{\frac{1+x^3}{(-4+x)^2\,(-3+x)\,x}}\left(1+\frac{x}{1+n^2}\right)\left(\frac{(1+x)^2}{(-4+x)\,(-3+x)^2\,\left(-5+x^2\right)^3}\right)^n\sqrt{-5+x^2}$$

```
Test[f, Der[x], {S[n]}]
```

$Aborted

```
Test[((z^2 - 1) / 2 / (z - x)) ^n * (1 - z) ^alpha * (1 + z) ^beta / (z - x), Der[z], {S[n]}]
```

$$\{0.3184,\ \{\{(16+16\,\text{alpha}+4\,\text{alpha}^2+16\,\text{beta}+8\,\text{alpha beta}+4\,\text{beta}^2+32\,n+20\,\text{alpha}\,n+2\,\text{alpha}^2\,n+20\,\text{beta}\,n+4\,\text{alpha beta}\,n+2\,\text{beta}^2\,n+20\,n^2+6\,\text{alpha}\,n^2+6\,\text{beta}\,n^2+4\,n^3)\,S_n^2+$$
$$(-3\,\text{alpha}^2-\text{alpha}^3-\text{alpha}^2\,\text{beta}+3\,\text{beta}^2+\text{alpha beta}^2+\text{beta}^3-2\,\text{alpha}^2\,n+2\,\text{beta}^2\,n-24\,x-26\,\text{alpha}\,x-9\,\text{alpha}^2\,x-\text{alpha}^3\,x-26\,\text{beta}\,x-18\,\text{alpha beta}\,x-3\,\text{alpha}^2\,\text{beta}\,x-9\,\text{beta}^2\,x-3\,\text{alpha beta}^2\,x-\text{beta}^3\,x-52\,n\,x-36\,\text{alpha}\,n\,x-6\,\text{alpha}^2\,n\,x-36\,\text{beta}\,n\,x-12\,\text{alpha beta}\,n\,x-6\,\text{beta}^2\,n\,x-36\,n^2\,x-12\,\text{alpha}\,n^2\,x-12\,\text{beta}\,n^2\,x-8\,n^3\,x)\,S_n+$$
$$(8+10\,\text{alpha}+2\,\text{alpha}^2+10\,\text{beta}+12\,\text{alpha beta}+2\,\text{alpha}^2\,\text{beta}+2\,\text{beta}^2+2\,\text{alpha beta}^2+20\,n+16\,\text{alpha}\,n+2\,\text{alpha}^2\,n+16\,\text{beta}\,n+8\,\text{alpha beta}\,n+2\,\text{beta}^2\,n+16\,n^2+6\,\text{alpha}\,n^2+6\,\text{beta}\,n^2+4\,n^3)\},$$
$$\{\{\frac{1}{2\,(x-z)}(-4-4\,\text{alpha}-\text{alpha}^2-4\,\text{beta}-2\,\text{alpha beta}-\text{beta}^2-6\,n-3\,\text{alpha}\,n-3\,\text{beta}\,n-2\,n^2-4\,\text{alpha}\,x-\text{alpha}^2\,x+4\,\text{beta}\,x+\text{beta}^2\,x-2\,\text{alpha}\,n\,x+2\,\text{beta}\,n\,x+4\,\text{alpha}\,z+\text{alpha}^2\,z-4\,\text{beta}\,z-\text{beta}^2\,z+2\,\text{alpha}\,n\,z-2\,\text{beta}\,n\,z+8\,x\,z+6\,\text{alpha}\,x\,z+\text{alpha}^2\,x\,z+6\,\text{beta}\,x\,z+2\,\text{alpha beta}\,x\,z+\text{beta}^2\,x\,z+12\,n\,x\,z+4\,\text{alpha}\,n\,x\,z+4\,\text{beta}\,n\,x\,z+4\,n^2\,x\,z+2\,\text{alpha}\,z^2+\text{alpha}^2\,z^2+2\,\text{beta}\,z^2+2\,\text{alpha beta}\,z^2+\text{beta}^2\,z^2+2\,\text{alpha}\,n\,z^2+2\,\text{beta}\,n\,z^2+4\,\text{alpha}\,x\,z^2+\text{alpha}^2\,x\,z^2-4\,\text{beta}\,x\,z^2-\text{beta}^2\,x\,z^2+2\,\text{alpha}\,n\,x\,z^2-2\,\text{beta}\,n\,x\,z^2-4\,\text{alpha}\,z^3-\text{alpha}^2\,z^3+4\,\text{beta}\,z^3+\text{beta}^2\,z^3-2\,\text{alpha}\,n\,z^3+2\,\text{beta}\,n\,z^3-8\,x\,z^3-6\,\text{alpha}\,x\,z^3-\text{alpha}^2\,x\,z^3-6\,\text{beta}\,x\,z^3-2\,\text{alpha beta}\,x\,z^3-\text{beta}^2\,x\,z^3-12\,n\,x\,z^3-4\,\text{alpha}\,n\,x\,z^3-4\,\text{beta}\,n\,x\,z^3-4\,n^2\,x\,z^3+4\,z^4+2\,\text{alpha}\,z^4+2\,\text{beta}\,z^4+6\,n\,z^4+\text{alpha}\,n\,z^4+\text{beta}\,n\,z^4+2\,n^2\,z^4)\}\}\}\}$$

```
Test[(1 + x) ^ (100 * n) / x ^ (n + 1), Der[x], {S[n]}]
```

$Aborted

```
f = 4 * (x * (12 * x + 10) ^2 / (87 * x + 37)) ^ (-n - 1) *
   x * (6 * x + 5) * (1044 * x ^2 + 666 * x + 185) / (87 * x + 37) ^2
```

$$\left(4\,x\,(5+6\,x)\left(\frac{x\,(10+12\,x)^2}{37+87\,x}\right)^{-1-n}\left(185+666\,x+1044\,x^2\right)\right)\Big/\,(37+87\,x)^2$$

```
Test[f, Der[x], {S[n]}]
```

$$\{0.336258, \{\{(28\,968\,000 + 244\,240\,000\, n + 255\,174\,000\, n^2 + 70\,148\,000\, n^3)\, S_n^2 + (521\,424 + 98\,781\,996\, n + 222\,482\,304\, n^2 + 104\,077\,896\, n^3)\, S_n + (-61\,679\,781\, n + 69\,142\,815\, n^2 + 108\,433\,494\, n^3)\},$$

$$\{\{(91\,707\,256\,500 + 773\,218\,045\,000\, n + 807\,833\,038\,875\, n^2 + 222\,075\,415\,250\, n^3 + 1\,458\,956\,818\,875\, x + 8\,163\,839\,566\,200\, n\, x + 8\,481\,824\,965\,275\, n^2\, x + 2\,457\,048\,480\,900\, n^3\, x + 7\,570\,839\,656\,925\, x^2 + 35\,954\,556\,193\,260\, n\, x^2 + 39\,151\,775\,566\,800\, n^2\, x^2 + 12\,516\,065\,579\,160\, n^3\, x^2 + 18\,649\,288\,387\,656\, x^3 + 87\,366\,431\,049\,066\, n\, x^3 + 103\,874\,052\,113\,856\, n^2\, x^3 + 37\,075\,911\,988\,986\, n^3\, x^3 + 24\,306\,010\,315\,344\, x^4 + 129\,682\,851\,318\,018\, n\, x^4 + 172\,427\,616\,936\,846\, n^2\, x^4 + 67\,813\,119\,407\,592\, n^3\, x^4 + 16\,295\,493\,624\,192\, x^5 + 118\,502\,478\,507\,744\, n\, x^5 + 177\,570\,940\,399\,128\, n^2\, x^5 + 75\,363\,955\,515\,576\, n^3\, x^5 + 4\,316\,597\,793\,504\, x^6 + 60\,389\,698\,114\,656\, n\, x^6 + 102\,026\,347\,610\,400\, n^2\, x^6 + 45\,953\,247\,289\,248\, n^3\, x^6 + 13\,909\,037\,334\,624\, n\, x^7 + 26\,135\,130\,650\,112\, n^2\, x^7 + 12\,226\,093\,315\,488\, n^3\, x^7) / ((1 + n)\, x\, (5 + 6\, x)^3\, (185 + 666\, x + 1044\, x^2))\}\}\}\}$$

```
f = 4 * (x * (76 * x^2 - 90 * x + 92) ^2 / (82 * x^2 + 58 * x + 65)) ^ (-n - 1) *
   x * (38 * x^2 - 45 * x + 46) *
   (9348 * x^4 + 5126 * x^3 + 3358 * x^2 - 8775 * x + 2990) / (82 * x^2 + 58 * x + 65) ^2
```

$$\left(4\, x\, (46 - 45\, x + 38\, x^2) \left(\frac{x\,(92 - 90\, x + 76\, x^2)^2}{65 + 58\, x + 82\, x^2}\right)^{-1-n} (2990 - 8775\, x + 3358\, x^2 + 5126\, x^3 + 9348\, x^4)\right) \Big/ (65 + 58\, x + 82\, x^2)^2$$

```
Test[f, Der[x], {S[n]}][[1]]
```

4.27648

```
f = (x * (33 * x^4 - 59 * x^3 - 9 * x^2 + 22 * x - 99) ^2 /
      (86 * x^4 - 37 * x^3 + 94 * x^2 + 80 * x - 31)) ^
    (-n - 1) * x * (33 * x^4 - 59 * x^3 - 9 * x^2 + 22 * x - 99) *
   (14190 * x^8 - 22548 * x^7 + 29672 * x^6 - 7836 * x^5 - 14523 * x^4 + 4665 * x^3 +
      14221 * x^2 - 2046 * x + 3069) / (86 * x^4 - 37 * x^3 + 94 * x^2 + 80 * x - 31) ^2
```

$$\left(x\,(-99 + 22\, x - 9\, x^2 - 59\, x^3 + 33\, x^4) \left((x\,(-99 + 22\, x - 9\, x^2 - 59\, x^3 + 33\, x^4)^2) / (-31 + 80\, x + 94\, x^2 - 37\, x^3 + 86\, x^4)\right)^{-1-n} (3069 - 2046\, x + 14\,221\, x^2 + 4665\, x^3 - 14\,523\, x^4 - 7836\, x^5 + 29\,672\, x^6 - 22\,548\, x^7 + 14\,190\, x^8)\right) \Big/ (-31 + 80\, x + 94\, x^2 - 37\, x^3 + 86\, x^4)^2$$

```
Test[f, Der[x], {S[n]}][[1]]
```

611.192

Integrals with Gegenbauer polynomials

```
f = x^l * GegenbauerC[m, mu, x] * GegenbauerC[n, nu, x] * (-x^2 + 1) ^ (nu - 1 / 2)
```

$$x^{l} \left(1-x^{2}\right)^{-\frac{1}{2}+\text{nu}} \text{GegenbauerC}[m, \text{mu}, x]\, \text{GegenbauerC}[n, \text{nu}, x]$$

```
Test[f /. l → 0, Der[x], {S[m], S[n], S[mu], S[nu]}]
```

$$\Big\{2.30963,$$

$$\Big\{\Big\{\big(-4\,\text{nu}+m^2\,\text{nu}+4\,\text{mu}\,\text{nu}+2\,m\,\text{mu}\,\text{nu}-4\,n\,\text{nu}+2\,\text{mu}\,n\,\text{nu}-n^2\,\text{nu}-8\,\text{nu}^2+4\,\text{mu}\,\text{nu}^2-4\,n\,\text{nu}^2-4\,\text{nu}^3\big)\,S_{\text{nu}}+\big(n-\text{mu}\,n+n^2-\text{mu}\,n^2+2\,\text{nu}-2\,\text{mu}\,\text{nu}+5\,n\,\text{nu}-4\,\text{mu}\,n\,\text{nu}+n^2\,\text{nu}+6\,\text{nu}^2-4\,\text{mu}\,\text{nu}^2+4\,n\,\text{nu}^2+4\,\text{nu}^3\big),$$

$$\big(-4\,\text{mu}^2+4\,\text{mu}\,\text{nu}\big)\,S_{\text{mu}}+\big(m^2+4\,m\,\text{mu}+4\,\text{mu}^2-n^2-2\,m\,\text{nu}-4\,\text{mu}\,\text{nu}-2\,n\,\text{nu}\big),$$

$$\big(8-2\,m^2-8\,\text{mu}-4\,m\,\text{mu}+20\,n-3\,m^2\,n-16\,\text{mu}\,n-6\,m\,\text{mu}\,n+18\,n^2-m^2\,n^2-10\,\text{mu}\,n^2-2\,m\,\text{mu}\,n^2+7\,n^3-2\,\text{mu}\,n^3+n^4+16\,\text{nu}-8\,\text{mu}\,\text{nu}+32\,n\,\text{nu}-12\,\text{mu}\,n\,\text{nu}+20\,n^2\,\text{nu}-4\,\text{mu}\,n^2\,\text{nu}+4\,n^3\,\text{nu}+8\,\text{nu}^2+12\,n\,\text{nu}^2+4\,n^2\,\text{nu}^2\big)\,S_n^2+$$

$$\big(m^2\,n+2\,m\,\text{mu}\,n+m^2\,n^2-2\,\text{mu}\,n^2+2\,m\,\text{mu}\,n^2-n^3-2\,\text{mu}\,n^3-n^4+2\,m^2\,\text{nu}+4\,m\,\text{mu}\,\text{nu}+4\,m^2\,n\,\text{nu}-4\,\text{mu}\,n\,\text{nu}+8\,m\,\text{mu}\,n\,\text{nu}-2\,n^2\,\text{nu}-8\,\text{mu}\,n^2\,\text{nu}-4\,n^3\,\text{nu}+4\,m^2\,\text{nu}^2+8\,m\,\text{mu}\,\text{nu}^2-8\,\text{mu}\,n\,\text{nu}^2-4\,n^2\,\text{nu}^2\big),$$

$$\big(2+m+3\,n+m\,n+n^2+2\,\text{nu}+2\,n\,\text{nu}\big)\,S_m\,S_n+\big(-m\,n-2\,\text{mu}\,n-n^2-2\,m\,\text{nu}-4\,\text{mu}\,\text{nu}-2\,n\,\text{nu}\big),$$

$$\big(-4-4\,m-m^2+n^2-4\,\text{nu}-2\,m\,\text{nu}+2\,n\,\text{nu}\big)\,S_m^2+$$

$$\big(m^2+4\,m\,\text{mu}+4\,\text{mu}^2-n^2-2\,m\,\text{nu}-4\,\text{mu}\,\text{nu}-2\,n\,\text{nu}\big)\Big\},\ \Big\{\Big\{-\frac{2\,\big(-\text{mu}\,\text{nu}+\text{mu}\,\text{nu}\,x^2\big)}{x}\,S_{\text{mu}}\,S_{\text{nu}}+$$

$$\frac{1}{x}\big(-\text{nu}-m\,\text{nu}-2\,\text{mu}\,\text{nu}-n\,\text{nu}-2\,\text{nu}^2+2\,\text{nu}\,x^2+m\,\text{nu}\,x^2+n\,\text{nu}\,x^2+2\,\text{nu}^2\,x^2\big)\,S_{\text{nu}}+$$

$$\frac{1}{2\,x}\big(n+n^2+2\,\text{nu}+4\,n\,\text{nu}+4\,\text{nu}^2-n\,x^2-n^2\,x^2-2\,\text{nu}\,x^2-4\,n\,\text{nu}\,x^2-4\,\text{nu}^2\,x^2\big)\Big\},$$

$$\Big\{-\frac{2\,\big(-\text{mu}+\text{mu}\,x^2\big)}{x}\,S_{\text{mu}}-\frac{2\,\text{nu}}{x}\,S_{\text{nu}}+\frac{1}{x}\big(-m-2\,\text{mu}+n+2\,\text{nu}+m\,x^2+2\,\text{mu}\,x^2-n\,x^2-2\,\text{nu}\,x^2\big)\Big\},$$

$$\Big\{-\frac{1}{x}8\,\big(-\text{mu}\,\text{nu}-\text{mu}\,n\,\text{nu}-\text{mu}\,\text{nu}^2+\text{mu}\,\text{nu}\,x^2+\text{mu}\,n\,\text{nu}\,x^2+\text{mu}\,\text{nu}^2\,x^2\big)\,S_{\text{mu}}\,S_{\text{nu}}+$$

$$\frac{1}{x}4\,\big(-\text{nu}-m\,\text{nu}-2\,\text{mu}\,\text{nu}-2\,n\,\text{nu}-m\,n\,\text{nu}-2\,\text{mu}\,n\,\text{nu}-n^2\,\text{nu}-3\,\text{nu}^2-m\,\text{nu}^2-2\,\text{mu}\,\text{nu}^2-3\,n\,\text{nu}^2-2\,\text{nu}^3+2\,\text{nu}\,x^2+m\,\text{nu}\,x^2+3\,n\,\text{nu}\,x^2+m\,n\,\text{nu}\,x^2+n^2\,\text{nu}\,x^2+4\,\text{nu}^2\,x^2+m\,\text{nu}^2\,x^2+3\,n\,\text{nu}^2\,x^2+2\,\text{nu}^3\,x^2\big)\,S_{\text{nu}}-$$

$$\frac{1}{x}2\,\big(-n-2\,n^2-n^3-2\,\text{nu}-7\,n\,\text{nu}-5\,n^2\,\text{nu}-6\,\text{nu}^2-8\,n\,\text{nu}^2-4\,\text{nu}^3+n\,x^2+2\,n^2\,x^2+n^3\,x^2+2\,\text{nu}\,x^2+7\,n\,\text{nu}\,x^2+5\,n^2\,\text{nu}\,x^2+6\,\text{nu}^2\,x^2+8\,n\,\text{nu}^2\,x^2+4\,\text{nu}^3\,x^2\big)\Big\},$$

$$\Big\{\frac{4\,\big(-\text{mu}\,\text{nu}+\text{mu}\,\text{nu}\,x^2\big)}{(1+m)\,x}\,S_{\text{mu}}\,S_{\text{nu}}+\frac{2\,(-\text{nu}+2\,\text{mu}\,\text{nu})}{(1+m)\,x}\,S_{\text{nu}}+\frac{n+2\,\text{nu}-n\,x^2-2\,\text{nu}\,x^2}{x}\Big\},$$

$$\Big\{\Big(\Big(8\,\big(-\text{mu}\,\text{nu}-m\,\text{mu}\,\text{nu}-\text{mu}^2\,\text{nu}+\text{mu}\,\text{nu}\,x^2+m\,\text{mu}\,\text{nu}\,x^2+\text{mu}^2\,\text{nu}\,x^2\big)\Big)\Big/\Big(\big(2+3\,m+m^2\big)\,x\Big)\Big)$$

$$S_{\text{mu}}\,S_{\text{nu}}+\frac{1}{\big(2+3\,m+m^2\big)\,x}4\,\big(2\,\text{mu}+3\,m\,\text{mu}+m^2\,\text{mu}+2\,\text{mu}^2+m\,\text{mu}^2+\text{mu}\,n+m\,\text{mu}\,n+$$

$$\text{mu}^2\,n+2\,\text{mu}\,\text{nu}+2\,m\,\text{mu}\,\text{nu}+2\,\text{mu}^2\,\text{nu}-4\,\text{mu}\,x^2-6\,m\,\text{mu}\,x^2-2\,m^2\,\text{mu}\,x^2-$$

```
      4 mu^2 x^2 - 2 m mu^2 x^2 - 2 mu n x^2 - 2 m mu n x^2 - 2 mu^2 n x^2 - 4 mu nu x^2 -
      4 m mu nu x^2 - 4 mu^2 nu x^2 + 2 mu x^4 + 3 m mu x^4 + m^2 mu x^4 + 2 mu^2 x^4 + m mu^2 x^4 +
      mu n x^4 + m mu n x^4 + mu^2 n x^4 + 2 mu nu x^4 + 2 m mu nu x^4 + 2 mu^2 nu x^4) S_mu +
  ((4 (-nu - m nu + mu nu + 2 m mu nu + 2 mu^2 nu)) / ((2 + 3 m + m^2) x)) S_nu + 1/((2 + 3 m + m^2) x)
   2 (-2 m - 3 m^2 - m^3 - 4 mu - 8 m mu - 3 m^2 mu - 4 mu^2 - 2 m mu^2 + n + m n - mu n - 2 m mu n -
     2 mu^2 n + 2 nu + 2 m nu - 2 mu nu - 4 m mu nu - 4 mu^2 nu + 2 m x^2 + 3 m^2 x^2 + m^3 x^2 +
     4 mu x^2 + 8 m mu x^2 + 3 m^2 mu x^2 + 4 mu^2 x^2 + 2 m mu^2 x^2 - n x^2 - m n x^2 + mu n x^2 + 2 m mu
      n x^2 + 2 mu^2 n x^2 - 2 nu x^2 - 2 m nu x^2 + 2 mu nu x^2 + 4 m mu nu x^2 + 4 mu^2 nu x^2)}}}}
```

```
Test[f, Der[x], {S[m], S[n], S[l], S[mu], S[nu]}]
```

```
{5.42463,
 {{(-64 nu - 32 l nu - 4 l^2 nu + 4 m^2 nu + 32 mu nu + 8 l mu nu + 8 m mu nu - 32 n nu - 8 l n nu +
      8 mu n nu - 4 n^2 nu - 128 nu^2 - 48 l nu^2 - 4 l^2 nu^2 + 4 m^2 nu^2 +
      48 mu nu^2 + 8 l mu nu^2 + 8 m mu nu^2 - 48 n nu^2 - 8 l n nu^2 + 8 mu n nu^2 -
      4 n^2 nu^2 - 80 nu^3 - 16 l nu^3 + 16 mu nu^3 - 16 n nu^3 - 16 nu^4) S_nu^2 +
    (-4 mu^2 n - 4 mu^2 n^2 - 8 mu^2 nu + 4 mu n nu - 16 mu^2 n nu + 4 mu n^2 nu +
      8 mu nu^2 - 16 mu^2 nu^2 + 16 mu n nu^2 + 16 mu nu^3) S_mu +
    (72 nu + 28 l nu + 2 l^2 nu - 6 m^2 nu - 48 mu nu - 12 l mu nu - 12 m mu nu + 80 n nu +
      20 l n nu - 4 m^2 n nu - 48 mu n nu - 8 l mu n nu - 8 m mu n nu + 30 n^2 nu +
      4 l n^2 nu - 12 mu n^2 nu + 4 n^3 nu + 184 nu^2 + 60 l nu^2 + 4 l^2 nu^2 - 8 m^2 nu^2 -
      96 mu nu^2 - 16 l mu nu^2 - 16 m mu nu^2 + 140 n nu^2 + 24 l n nu^2 - 48 mu n nu^2 +
      28 n^2 nu^2 + 160 nu^3 + 32 l nu^3 - 48 mu nu^3 + 64 n nu^3 + 48 nu^4) S_nu +
    (-6 n - l n + m^2 n + 6 mu n + 4 m mu n + 4 mu^2 n - 10 n^2 - l n^2 + m^2 n^2 + 10 mu n^2 + 4 m mu n^2 +
      4 mu^2 n^2 - 5 n^3 + 4 mu n^3 - n^4 - 12 nu - 2 l nu + 2 m^2 nu + 12 mu nu + 8 m mu nu + 8 mu^2 nu -
      46 n nu - 6 l n nu - 2 m n nu + 4 m^2 n nu + 36 mu n nu + 16 m mu n nu + 16 mu^2 n nu -
      38 n^2 nu - 2 l n^2 nu - 2 m n^2 nu + 20 mu n^2 nu - 10 n^3 nu - 52 nu^2 - 8 l nu^2 - 4 m nu^2 +
      4 m^2 nu^2 + 32 mu nu^2 + 16 m mu nu^2 + 16 mu^2 nu^2 - 92 n nu^2 - 8 l n nu^2 - 8 m n nu^2 +
      32 mu n nu^2 - 36 n^2 nu^2 - 72 nu^3 - 8 l nu^3 - 8 m nu^3 + 16 mu nu^3 - 56 n nu^3 - 32 nu^4),
   -4 mu nu S_mu S_nu + (-4 nu - 2 l nu + 2 m nu + 4 mu nu - 2 n nu - 4 nu^2) S_nu +
    (n + n^2 + 2 nu + 4 n nu + 4 nu^2),
   (8 mu + 16 mu^2 + 8 mu^3 - 8 mu nu - 8 mu^2 nu) S_mu^2 +
    (-8 mu + 2 l mu - 8 m mu - 2 m^2 mu - 20 mu^2 + 4 l mu^2 - 12 m mu^2 - 16 mu^3 -
      4 mu^2 n + 2 mu n^2 + 12 mu nu + 8 m mu nu + 8 mu^2 nu + 8 mu n nu + 8 mu nu^2) S_mu +
    (8 nu + 8 l nu + 2 l^2 nu - 2 m^2 nu - 8 mu nu - 4 l mu nu - 4 m mu nu + 8 n nu +
      4 l n nu - 4 mu n nu + 2 n^2 nu + 16 nu^2 + 8 l nu^2 - 8 mu nu^2 + 8 n nu^2 + 8 nu^3) S_nu +
    (-l m + m^2 - l m^2 + m^3 - 2 l mu + 4 m mu - 4 l m mu + 6 m^2 mu + 4 mu^2 - 4 l mu^2 + 12 m mu^2 + 8 mu^3 -
      2 n - l n + m^2 n + 2 mu n + 4 m mu n + 4 mu^2 n - 3 n^2 - l n^2 - m n^2 - n^3 - 4 nu - 2 l nu - 2 m nu -
      12 n nu - 4 l n nu - 4 m n nu - 6 n^2 nu - 12 nu^2 - 4 l nu^2 - 4 m nu^2 - 12 n nu^2 - 8 nu^3),
   (-18 nu - 12 l nu - 2 l^2 nu + 2 m^2 nu + 12 mu nu + 4 l mu nu + 4 m mu nu - 12 n nu -
      4 l n nu + 4 mu n nu - 2 n^2 nu - 24 nu^2 - 8 l nu^2 + 8 mu nu^2 - 8 n nu^2 - 8 nu^3) S_l S_nu +
    (n + l n + m n + l m n + 2 nu + 2 l nu + 2 m nu + 2 l m nu) S_m +
    (-2 - 3 l - l^2 - 3 n - 4 l n - l^2 n - n^2 - l n^2 - 2 nu - 2 l nu - 2 n nu - 2 l n nu) S_n +
```

$\left(5\,n + 4\,l\,n + l^2\,n - m\,n - l\,m\,n - 4\,mu\,n - 2\,l\,mu\,n + 3\,n^2 + l\,n^2 - 2\,mu\,n^2 + 10\,nu +\right.$
$8\,l\,nu + 2\,l^2\,nu - 2\,m\,nu - 2\,l\,m\,nu - 8\,mu\,nu - 4\,l\,mu\,nu + 16\,n\,nu + 6\,l\,n\,nu -$
$\left.8\,mu\,n\,nu + 2\,n^2\,nu + 20\,nu^2 + 8\,l\,nu^2 - 8\,mu\,nu^2 + 8\,n\,nu^2 + 8\,nu^3\right)\,S_l,$

$\left(-4\,mu^2 + 4\,mu\,nu\right)\,S_l\,S_{mu} + (1 + l + m + l\,m)\,S_m + (-1 - l - n - l\,n)\,S_n +$
$\left(-m - l\,m + m^2 - 2\,mu - 2\,l\,mu + 4\,m\,mu + 4\,mu^2 + n +\right.$
$\left.l\,n - n^2 + 2\,nu + 2\,l\,nu - 2\,m\,nu - 4\,mu\,nu - 2\,n\,nu\right)\,S_l,$

$\left(4 + 4\,l + l^2 - m^2 - 4\,mu - 2\,l\,mu - 2\,m\,mu - 4\,n - 2\,l\,n + 2\,mu\,n + n^2\right)\,S_l^2 +$
$\left(4\,mu^2 - 4\,mu\,nu\right)\,S_{mu} + \left(-8\,nu - 4\,l\,nu + 4\,mu\,nu - 4\,nu^2\right)\,S_{nu} + \left(-2 - 3\,l - l^2 + 2\,mu +\right.$
$\left.2\,l\,mu - 2\,m\,mu - 4\,mu^2 + 4\,n + 2\,l\,n - 2\,mu\,n + 6\,nu + 2\,l\,nu + 2\,m\,nu + 2\,n\,nu + 4\,nu^2\right),$

$\left(18\,nu + 12\,l\,nu + 2\,l^2\,nu - 2\,m^2\,nu - 12\,mu\,nu - 4\,l\,mu\,nu - 4\,m\,mu\,nu + 30\,n\,nu +\right.$
$16\,l\,n\,nu + 2\,l^2\,n\,nu - 2\,m^2\,n\,nu - 16\,mu\,n\,nu - 4\,l\,mu\,n\,nu - 4\,m\,mu\,n\,nu +$
$14\,n^2\,nu + 4\,l\,n^2\,nu - 4\,mu\,n^2\,nu + 2\,n^3\,nu + 24\,nu^2 + 8\,l\,nu^2 - 8\,mu\,nu^2 +$
$\left.32\,n\,nu^2 + 8\,l\,n\,nu^2 - 8\,mu\,n\,nu^2 + 8\,n^2\,nu^2 + 8\,nu^3 + 8\,n\,nu^3\right)\,S_n\,S_{nu} +$
$\left(-2\,n - 2\,l\,n - 2\,m\,n - 2\,l\,m\,n - 2\,n^2 - 2\,l\,n^2 - 2\,m\,n^2 - 2\,l\,m\,n^2 - 4\,nu - 4\,l\,nu - 4\,m\,nu -\right.$
$\left.4\,l\,m\,nu - 6\,n\,nu - 6\,l\,n\,nu - 6\,m\,n\,nu - 6\,l\,m\,n\,nu - 4\,nu^2 - 4\,l\,nu^2 - 4\,m\,nu^2 - 4\,l\,m\,nu^2\right)\,S_m +$
$\left(-5 + l^2 + m^2 + 6\,mu + 2\,l\,mu + 2\,m\,mu - 14\,n + 2\,l^2\,n + 2\,m^2\,n + 14\,mu\,n + 4\,l\,mu\,n +\right.$
$4\,m\,mu\,n - 14\,n^2 + l^2\,n^2 + m^2\,n^2 + 10\,mu\,n^2 + 2\,l\,mu\,n^2 + 2\,m\,mu\,n^2 - 6\,n^3 + 2\,mu\,n^3 - n^4 -$
$22\,nu - 6\,l\,nu + 2\,m^2\,nu + 16\,mu\,nu + 4\,l\,mu\,nu + 4\,m\,mu\,nu - 44\,n\,nu - 8\,l\,n\,nu + 2\,m^2\,n\,nu +$
$24\,mu\,n\,nu + 4\,l\,mu\,n\,nu + 4\,m\,mu\,n\,nu - 28\,n^2\,nu - 2\,l\,n^2\,nu + 8\,mu\,n^2\,nu - 6\,n^3\,nu - 24\,nu^2 -$
$\left.4\,l\,nu^2 + 8\,mu\,nu^2 - 36\,n\,nu^2 - 4\,l\,n\,nu^2 + 8\,mu\,n\,nu^2 - 12\,n^2\,nu^2 - 8\,nu^3 - 8\,n\,nu^3\right)\,S_n +$
$\left(-n - 2\,l\,n - l^2\,n + 2\,m\,n + 2\,l\,m\,n - m^2\,n + 2\,mu\,n + 2\,l\,mu\,n - 2\,m\,mu\,n - n^2 - 2\,l\,n^2 -\right.$
$l^2\,n^2 + 2\,m\,n^2 + 2\,l\,m\,n^2 - m^2\,n^2 + 4\,mu\,n^2 + 2\,l\,mu\,n^2 - 2\,m\,mu\,n^2 + n^3 + 2\,mu\,n^3 +$
$n^4 - 2\,nu - 4\,l\,nu - 2\,l^2\,nu + 4\,m\,nu + 4\,l\,m\,nu - 2\,m^2\,nu + 4\,mu\,nu + 4\,l\,mu\,nu -$
$4\,m\,mu\,nu - 2\,n\,nu - 4\,l\,n\,nu - 2\,l^2\,n\,nu + 6\,m\,n\,nu + 6\,l\,m\,n\,nu - 4\,m^2\,n\,nu +$
$8\,mu\,n\,nu + 4\,l\,mu\,n\,nu - 8\,m\,mu\,n\,nu - 2\,l\,n^2\,nu + 8\,mu\,n^2\,nu + 4\,n^3\,nu + 4\,m\,nu^2 +$
$\left.4\,l\,m\,nu^2 - 4\,m^2\,nu^2 - 8\,m\,mu\,nu^2 - 4\,n\,nu^2 - 4\,l\,n\,nu^2 + 8\,mu\,n\,nu^2 + 4\,n^2\,nu^2\right)\,S_l,$

$\left(-4\,mu^2 - 4\,mu^2\,n + 4\,mu\,nu + 4\,mu\,n\,nu\right)\,S_n\,S_{mu} +$
$(n + l\,n + m\,n + l\,m\,n + 2\,nu + 2\,l\,nu + 2\,m\,nu + 2\,l\,m\,nu)\,S_m +$
$\left(-2\,mu - 2\,l\,mu + 2\,m\,mu + 4\,mu^2 - n - l\,n - 2\,mu\,n - 2\,l\,mu\,n + 2\,m\,mu\,n +\right.$
$\left.4\,mu^2\,n - n^2 - l\,n^2 - 2\,m\,nu - 4\,mu\,nu - 2\,m\,n\,nu - 4\,mu\,n\,nu\right)\,S_n +$
$\left(-m\,n - l\,m\,n + m^2\,n + 2\,m\,mu\,n + n^2 + l\,n^2 - 2\,mu\,n^2 - n^3 - 2\,m\,nu - 2\,l\,m\,nu + 2\,m^2\,nu + 4\,m\,mu\,nu +\right.$
$\left.2\,n\,nu + 2\,l\,n\,nu - 4\,mu\,n\,nu - 2\,n^2\,nu\right)\,S_l,\ (1 + n)\,S_n\,S_l + 2\,nu\,S_{nu} + (-n - 2\,nu),$

$\left(2 + 3\,n + n^2\right)\,S_n^2 + \left(4\,nu + 4\,n\,nu + 4\,nu^2\right)\,S_{nu} + \left(-n - n^2 - 2\,nu - 4\,n\,nu - 4\,nu^2\right),$

$\left(-6\,nu - 2\,l\,nu - 2\,m\,nu - 2\,n\,nu - 4\,nu^2\right)\,S_m\,S_{nu} +$
$\left(2\,n + m\,n + n^2 + 4\,nu + 2\,m\,nu + 4\,n\,nu + 4\,nu^2\right)\,S_m + (-1 - l - n - l\,n)\,S_n +$
$\left(n + l\,n - m\,n - 2\,mu\,n - n^2 + 2\,nu + 2\,l\,nu - 2\,m\,nu - 4\,mu\,nu - 2\,n\,nu\right)\,S_l,$

$\left(-4\,mu^2 + 4\,mu\,nu\right)\,S_m\,S_{mu} + \left(1 + l + m + l\,m + 2\,mu + 2\,m\,mu + 4\,mu^2 - 2\,nu - 2\,m\,nu - 4\,mu\,nu\right)\,S_m +$
$(-1 - l - n - l\,n)\,S_n + \left(-m - l\,m + m^2 - 2\,mu - 2\,l\,mu + 4\,m\,mu +\right.$
$\left.4\,mu^2 + n + l\,n - n^2 + 2\,nu + 2\,l\,nu - 2\,m\,nu - 4\,mu\,nu - 2\,n\,nu\right)\,S_l,$

$\left(-2 - l - 3\,m - l\,m - m^2 + n + m\,n\right)\,S_m\,S_l + \left(4\,mu^2 - 4\,mu\,nu\right)\,S_{mu} +$
$\left(-4\,nu - 2\,l\,nu + 2\,m\,nu + 4\,mu\,nu - 2\,n\,nu - 4\,nu^2\right)\,S_{nu} +$
$\left(m + l\,m + 2\,mu + 2\,l\,mu - 2\,m\,mu - 4\,mu^2 + n - m\,n - 2\,mu\,n + n^2 + 2\,nu + 4\,n\,nu + 4\,nu^2\right),$

$(1 + l + m + l\,m + n + l\,n + m\,n + l\,m\,n)\,S_m\,S_n + \left(-4\,mu^2\,n - 8\,mu^2\,nu + 4\,mu\,n\,nu + 8\,mu\,nu^2\right)\,S_{mu} +$

$(4\,nu + 2\,l\,nu + 2\,m\,nu + 2\,l\,m\,nu - 2\,m^2\,nu - 4\,mu\,nu - 4\,m\,mu\,nu + 6\,n\,nu + 2\,l\,n\,nu -$
$4\,mu\,n\,nu + 2\,n^2\,nu + 12\,nu^2 + 4\,l\,nu^2 - 8\,mu\,nu^2 + 8\,n\,nu^2 + 8\,nu^3)\,S_{nu} +$
$(-n - m\,n - l\,m\,n + m^2\,n - 2\,l\,mu\,n + 4\,m\,mu\,n + 4\,mu^2\,n - 2\,n^2 + 2\,mu\,n^2 - n^3 - 2\,nu -$
$2\,m\,nu - 2\,l\,m\,nu + 2\,m^2\,nu - 4\,l\,mu\,nu + 8\,m\,mu\,nu + 8\,mu^2\,nu - 8\,n\,nu -$
$2\,m\,n\,nu + 4\,mu\,n\,nu - 6\,n^2\,nu - 8\,nu^2 - 4\,m\,nu^2 - 12\,n\,nu^2 - 8\,nu^3),$
$(-4 - 2\,l - 8\,m - 3\,l\,m - 5\,m^2 - l\,m^2 - m^3 + 2\,n + 3\,m\,n + m^2\,n)\,S_m^2 +$
$(8\,mu^2 + 8\,m\,mu^2 + 8\,mu^3 - 8\,mu\,nu - 8\,m\,mu\,nu - 8\,mu^2\,nu)\,S_{mu} +$
$(-8\,nu - 4\,l\,nu - 4\,m\,nu - 4\,l\,m\,nu + 4\,m^2\,nu - 4\,l\,mu\,nu + 12\,m\,mu\,nu +$
$8\,mu^2\,nu - 4\,n\,nu - 4\,m\,n\,nu - 4\,mu\,n\,nu - 8\,nu^2 - 8\,m\,nu^2 - 8\,mu\,nu^2)\,S_{nu} +$
$(l\,m - m^2 + l\,m^2 - m^3 + 2\,l\,mu - 4\,m\,mu + 4\,l\,m\,mu - 6\,m^2\,mu - 4\,mu^2 + 4\,l\,mu^2 - 12\,m\,mu^2 -$
$8\,mu^3 + 2\,n + m\,n - m^2\,n - 4\,m\,mu\,n - 4\,mu^2\,n + 2\,n^2 + 2\,m\,n^2 + 2\,mu\,n^2 + 4\,nu +$
$4\,m\,nu + 4\,mu\,nu + 8\,n\,nu + 8\,m\,n\,nu + 8\,mu\,n\,nu + 8\,nu^2 + 8\,m\,nu^2 + 8\,mu\,nu^2)\},$
$\{\{8\,(-mu\,nu\,x - mu\,n\,nu\,x - mu\,nu^2\,x + mu\,nu\,x^3 + mu\,n\,nu\,x^3 + mu\,nu^2\,x^3)\,S_{mu}\,S_{nu} -$
$2\,(-mu\,n\,x - mu\,n^2\,x - 2\,mu\,nu\,x - 4\,mu\,n\,nu\,x - 4\,mu\,nu^2\,x + mu\,n\,x^3 + mu\,n^2\,x^3 + 2\,mu\,nu\,x^3 +$
$4\,mu\,n\,nu\,x^3 + 4\,mu\,nu^2\,x^3)\,S_{mu} - 2\,(-6\,nu\,x - 2\,l\,nu\,x - 2\,m\,nu\,x - 4\,mu\,nu\,x - 7\,n\,nu\,x -$
$2\,l\,n\,nu\,x - 2\,m\,n\,nu\,x - 4\,mu\,n\,nu\,x - n^2\,nu\,x - 8\,nu^2\,x - 2\,l\,nu^2\,x - 2\,m\,nu^2\,x -$
$4\,mu\,nu^2\,x - 2\,n\,nu^2\,x + 8\,nu\,x^3 + 2\,l\,nu\,x^3 + 2\,m\,nu\,x^3 + 10\,n\,nu\,x^3 + 2\,l\,n\,nu\,x^3 +$
$2\,m\,n\,nu\,x^3 + 2\,n^2\,nu\,x^3 + 12\,nu^2\,x^3 + 2\,l\,nu^2\,x^3 + 2\,m\,nu^2\,x^3 + 6\,n\,nu^2\,x^3 + 4\,nu^3\,x^3)\,S_{nu} +$
$(-4\,n\,x - l\,n\,x - m\,n\,x - 5\,n^2\,x - l\,n^2\,x - m\,n^2\,x - n^3\,x - 8\,nu\,x - 2\,l\,nu\,x - 2\,m\,nu\,x -$
$20\,n\,nu\,x - 4\,l\,n\,nu\,x - 4\,m\,n\,nu\,x - 6\,n^2\,nu\,x - 20\,nu^2\,x - 4\,l\,nu^2\,x - 4\,m\,nu^2\,x -$
$12\,n\,nu^2\,x - 8\,nu^3\,x + 4\,n\,x^3 + l\,n\,x^3 + m\,n\,x^3 + 5\,n^2\,x^3 + l\,n^2\,x^3 + m\,n^2\,x^3 + n^3\,x^3 +$
$8\,nu\,x^3 + 2\,l\,nu\,x^3 + 2\,m\,nu\,x^3 + 20\,n\,nu\,x^3 + 4\,l\,n\,nu\,x^3 + 4\,m\,n\,nu\,x^3 + 6\,n^2\,nu\,x^3 +$
$20\,nu^2\,x^3 + 4\,l\,nu^2\,x^3 + 4\,m\,nu^2\,x^3 + 12\,n\,nu^2\,x^3 + 8\,nu^3\,x^3)\},\ \{2\,nu\,x\,S_{nu}\},$
$\{4\,mu\,nu\,x\,S_{mu}\,S_{nu} - 2\,(mu + 2\,mu^2)\,x\,S_{mu} - 2\,(2\,nu + l\,nu + m\,nu + n\,nu + 2\,nu^2)\,x\,S_{nu} +$
$(m + m^2 + 2\,mu + 4\,m\,mu + 4\,mu^2)\,x\},$
$\{-4\,(-mu\,nu + mu\,nu\,x^2)\,S_{mu}\,S_{nu} + 2\,(-2\,nu - l\,nu - m\,nu - 2\,mu\,nu - n\,nu - 2\,nu^2 +$
$3\,nu\,x^2 + l\,nu\,x^2 + m\,nu\,x^2 + n\,nu\,x^2 + 2\,nu^2\,x^2)\,S_{nu} + (2\,n + l\,n + n^2 + 4\,nu + 2\,l\,nu +$
$4\,n\,nu + 4\,nu^2 - 2\,n\,x^2 - l\,n\,x^2 - n^2\,x^2 - 4\,nu\,x^2 - 2\,l\,nu\,x^2 - 4\,n\,nu\,x^2 - 4\,nu^2\,x^2)\},$
$\{-2\,(-mu + mu\,x^2)\,S_{mu} - 2\,nu\,S_{nu} + (-m - 2\,mu + n + 2\,nu + m\,x^2 + 2\,mu\,x^2 - n\,x^2 - 2\,nu\,x^2)\},$
$\{2\,(-mu\,x + mu\,x^3)\,S_{mu} + 2\,nu\,x\,S_{nu} + (2\,x + l\,x + m\,x - n\,x - 2\,x^3 - l\,x^3 - m\,x^3 + n\,x^3)\},$
$\{8\,(-mu\,nu - mu\,n\,nu - mu\,nu^2 + mu\,nu\,x^2 + mu\,n\,nu\,x^2 + mu\,nu^2\,x^2)\,S_{mu}\,S_{nu} -$
$4\,(-2\,nu - l\,nu - m\,nu - 2\,mu\,nu - 3\,n\,nu - l\,n\,nu - m\,n\,nu - 2\,mu\,n\,nu - n^2\,nu - 4\,nu^2 - l\,nu^2 -$
$m\,nu^2 - 2\,mu\,nu^2 - 3\,n\,nu^2 - 2\,nu^3 + 3\,nu\,x^2 + l\,nu\,x^2 + m\,nu\,x^2 + 4\,n\,nu\,x^2 + l\,n\,nu\,x^2 +$
$m\,n\,nu\,x^2 + n^2\,nu\,x^2 + 5\,nu^2\,x^2 + l\,nu^2\,x^2 + m\,nu^2\,x^2 + 3\,n\,nu^2\,x^2 + 2\,nu^3\,x^2)\,S_{nu} +$
$2\,(-2\,n - l\,n - 3\,n^2 - l\,n^2 - n^3 - 4\,nu - 2\,l\,nu - 10\,n\,nu - 3\,l\,n\,nu - 5\,n^2\,nu - 8\,nu^2 -$
$2\,l\,nu^2 - 8\,n\,nu^2 - 4\,nu^3 + 2\,n\,x^2 + l\,n\,x^2 + 3\,n^2\,x^2 + l\,n^2\,x^2 + n^3\,x^2 + 4\,nu\,x^2 + 2\,l\,nu\,x^2 +$
$10\,n\,nu\,x^2 + 3\,l\,n\,nu\,x^2 + 5\,n^2\,nu\,x^2 + 8\,nu^2\,x^2 + 2\,l\,nu^2\,x^2 + 8\,n\,nu^2\,x^2 + 4\,nu^3\,x^2)\},$
$\{-2\,(-mu\,n - 2\,mu\,nu + mu\,n\,x^2 + 2\,mu\,nu\,x^2)\,S_{mu} - 2\,(2\,mu\,nu + n\,nu)\,S_{nu} +$
$(-m\,n + n^2 - 2\,m\,nu + 2\,n\,nu + m\,n\,x^2 - n^2\,x^2 + 2\,m\,nu\,x^2 - 2\,n\,nu\,x^2)\},\ \{0\},\ \{0\},$
$\left\{\frac{4\,(-mu\,nu + mu\,nu\,x^2)}{1 + m}\,S_{mu}\,S_{nu} + \frac{2\,(-nu + 2\,mu\,nu)}{1 + m}\,S_{nu} + (n + 2\,nu - n\,x^2 - 2\,nu\,x^2)\right\},$
$\{-2\,(-mu + mu\,x^2)\,S_{mu} - 2\,nu\,S_{nu} + (-m - 2\,mu + n + 2\,nu + m\,x^2 + 2\,mu\,x^2 - n\,x^2 - 2\,nu\,x^2)\},$
$\{2\,(-mu\,x + mu\,x^3)\,S_{mu} + 2\,nu\,x\,S_{nu}\},$

$$\left\{\frac{4\left(-mu\,nu+mu\,nu\,x^2\right)}{x}S_{mu}S_{nu}-\frac{1}{x}2\left(-mu\,n-2\,mu\,nu+mu\,n\,x^2+2\,mu\,nu\,x^2\right)S_{mu}-\frac{1}{x}2\left(-m\,nu-2\,mu\,nu+nu\,x^2+m\,nu\,x^2+n\,nu\,x^2+2\,nu^2\,x^2\right)S_{nu}+\frac{1}{x}\left(-m\,n-2\,mu\,n-2\,m\,nu-4\,mu\,nu+m\,n\,x^2+2\,mu\,n\,x^2+2\,m\,nu\,x^2+4\,mu\,nu\,x^2\right)\right\},$$
$$\left\{4\left(-mu\,x-m\,mu\,x-mu^2\,x+mu\,x^3+m\,mu\,x^3+mu^2\,x^3\right)S_{mu}+4\,(nu+m\,nu+mu\,nu)\,x\,S_{nu}\right\}\}\}\}$$

```
F = (x + a) ^ (g + lambda - 1) * (a - x) ^ (beta - 1) *
  GegenbauerC[m, g, x / a] * GegenbauerC[n, lambda, x / a]
```

$$(a-x)^{-1+beta}\,(a+x)^{-1+g+lambda}\,\text{GegenbauerC}\left[m,g,\frac{x}{a}\right]\,\text{GegenbauerC}\left[n,lambda,\frac{x}{a}\right]$$

```
Test[f, Der[x], {S[m], S[n], Der[a], S[beta]}]
```

$$\{2.23597,\ \{\{S_{beta}-1,\ D_a,$$
$$\left(4\,n+2\,l\,n+4\,m\,n+l\,m\,n+m^2\,n+2\,n^2+2\,l\,n^2+3\,m\,n^2+l\,m\,n^2+m^2\,n^2-2\,n^3-m\,n^3+8\,nu+4\,l\,nu+8\,m\,nu+2\,l\,m\,nu+2\,m^2\,nu+8\,n\,nu+6\,l\,n\,nu+10\,m\,n\,nu+3\,l\,m\,n\,nu+3\,m^2\,n\,nu-6\,n^2\,nu-3\,m\,n^2\,nu+8\,nu^2+4\,l\,nu^2+8\,m\,nu^2+2\,l\,m\,nu^2+2\,m^2\,nu^2-4\,n\,nu^2-2\,m\,n\,nu^2\right)S_m^2+$$
$$\left(-2-2\,l-2\,m-2\,l\,m-2\,mu-2\,l\,mu-4\,n-4\,l\,n-4\,m\,n-4\,l\,m\,n-4\,mu\,n-4\,l\,mu\,n-2\,n^2-2\,l\,n^2-2\,m\,n^2-2\,l\,m\,n^2-2\,mu\,n^2-2\,l\,mu\,n^2-2\,nu-2\,l\,nu-2\,m\,nu-2\,l\,m\,nu-2\,mu\,nu-2\,l\,mu\,nu-2\,n\,nu-2\,l\,n\,nu-2\,m\,n\,nu-2\,l\,m\,n\,nu-2\,mu\,n\,nu-2\,l\,mu\,n\,nu\right)S_m\,S_n+$$
$$\left(4+2\,l+2\,m+2\,l\,m-2\,m^2+2\,l\,mu-6\,m\,mu-4\,mu^2+8\,n+3\,l\,n+5\,m\,n+3\,l\,m\,n-3\,m^2\,n+2\,mu\,n+3\,l\,mu\,n-9\,m\,mu\,n-6\,mu^2\,n+5\,n^2+l\,n^2+4\,m\,n^2+l\,m\,n^2-m^2\,n^2+3\,mu\,n^2+l\,mu\,n^2-3\,m\,mu\,n^2-2\,mu^2\,n^2+n^3+m\,n^3+mu\,n^3+4\,nu+4\,m\,nu+4\,mu\,nu+6\,n\,nu+6\,m\,n\,nu+6\,mu\,n\,nu+2\,n^2\,nu+2\,m\,n^2\,nu+2\,mu\,n^2\,nu\right)S_n^2+$$
$$\left(-l\,n+m\,n+2\,mu\,n+l\,mu\,n-m\,mu\,n-2\,mu^2\,n+n^2-l\,n^2+m\,n^2+mu\,n^2+l\,mu\,n^2-m\,mu\,n^2-2\,mu^2\,n^2+n^3-mu\,n^3-2\,l\,nu+2\,m\,nu+4\,mu\,nu+2\,l\,mu\,nu-2\,m\,mu\,nu-4\,mu^2\,nu+2\,n\,nu-4\,l\,n\,nu+4\,m\,n\,nu-l\,m\,n\,nu+m^2\,n\,nu+6\,mu\,n\,nu+2\,l\,mu\,n\,nu-4\,mu^2\,n\,nu+4\,n^2\,nu+m\,n^2\,nu-2\,mu\,n^2\,nu-4\,l\,nu^2+4\,m\,nu^2-2\,l\,m\,nu^2+2\,m^2\,nu^2+8\,mu\,nu^2+4\,m\,mu\,nu^2+4\,n\,nu^2+2\,m\,n\,nu^2\right),$$
$$\left(4+4\,l-4\,m+12\,n+8\,l\,n-8\,m\,n+13\,n^2+5\,l\,n^2-5\,m\,n^2+6\,n^3+l\,n^3-m\,n^3+n^4+2\,nu+2\,l\,nu-10\,m\,nu+13\,n\,nu+3\,l\,n\,nu-11\,m\,n\,nu+12\,n^2\,nu+l\,n^2\,nu-3\,m\,n^2\,nu+3\,n^3\,nu-4\,m\,nu^2+4\,n\,nu^2-2\,m\,n\,nu^2+2\,n^2\,nu^2\right)S_m\,S_n^2+$$
$$\left(-18-6\,l+6\,m+12\,mu-39\,n-11\,l\,n+11\,m\,n+22\,mu\,n-29\,n^2-6\,l\,n^2+6\,m\,n^2+12\,mu\,n^2-9\,n^3-l\,n^3+m\,n^3+2\,mu\,n^3-n^4-30\,nu-6\,l\,nu+6\,m\,nu+12\,mu\,nu-43\,n\,nu-5\,l\,n\,nu+5\,m\,n\,nu+10\,mu\,n\,nu-20\,n^2\,nu-l\,n^2\,nu+m\,n^2\,nu+2\,mu\,n^2\,nu-3\,n^3\,nu-12\,nu^2-10\,n\,nu^2-2\,n^2\,nu^2\right)S_n^3+$$
$$\left(-2\,n-2\,l\,n-2\,m\,n-n^2-3\,l\,n^2-3\,m\,n^2+2\,n^3-l\,n^3-m\,n^3+n^4-4\,nu-4\,l\,nu-4\,m\,nu-7\,n\,nu-11\,l\,n\,nu-11\,m\,n\,nu+6\,n^2\,nu-5\,l\,n^2\,nu-5\,m\,n^2\,nu+5\,n^3\,nu-10\,nu^2-10\,l\,nu^2-10\,m\,nu^2+2\,n\,nu^2-8\,l\,n\,nu^2-8\,m\,n\,nu^2+8\,n^2\,nu^2-4\,nu^3-4\,l\,nu^3-4\,m\,nu^3+4\,n\,nu^3\right)S_m+$$
$$\left(-2+2\,l+2\,m+4\,mu-7\,n+5\,l\,n+5\,m\,n+10\,mu\,n-9\,n^2+4\,l\,n^2+4\,m\,n^2+8\,mu\,n^2-5\,n^3+l\,n^3+m\,n^3+2\,mu\,n^3-n^4-8\,nu+8\,l\,nu+6\,m\,nu+12\,mu\,nu-21\,n\,nu+13\,l\,n\,nu+9\,m\,n\,nu+18\,mu\,n\,nu-18\,n^2\,nu+5\,l\,n^2\,nu+3\,m\,n^2\,nu+6\,mu\,n^2\,nu-5\,n^3\,nu-10\,nu^2+10\,l\,nu^2+4\,m\,nu^2+8\,mu\,nu^2-18\,n\,nu^2+8\,l\,n\,nu^2+2\,m\,n\,nu^2+4\,mu\,n\,nu^2-8\,n^2\,nu^2-4\,nu^3+4\,l\,nu^3-4\,n\,nu^3\right)S_n,$$
$$\left(384+192\,l+24\,l^2-24\,m^2-192\,mu-48\,l\,mu-48\,m\,mu+992\,n+448\,l\,n+50\,l^2\,n-\right.$$

$$50\, m^2\, n - 448\, mu\, n - 100\, l\, mu\, n - 100\, m\, mu\, n + 984\, n^2 + 380\, l\, n^2 + 35\, l^2\, n^2 - 35\, m^2\, n^2 -$$
$$380\, mu\, n^2 - 70\, l\, mu\, n^2 - 70\, m\, mu\, n^2 + 490\, n^3 + 150\, l\, n^3 + 10\, l^2\, n^3 - 10\, m^2\, n^3 -$$
$$150\, mu\, n^3 - 20\, l\, mu\, n^3 - 20\, m\, mu\, n^3 + 131\, n^4 + 28\, l\, n^4 + l^2\, n^4 - m^2\, n^4 - 28\, mu\, n^4 -$$
$$2\, l\, mu\, n^4 - 2\, m\, mu\, n^4 + 18\, n^5 + 2\, l\, n^5 - 2\, mu\, n^5 + n^6 + 768\, nu + 288\, l\, nu + 24\, l^2\, nu -$$
$$24\, m^2\, nu - 288\, mu\, nu - 48\, l\, mu\, nu - 48\, m\, mu\, nu + 1504\, n\, nu + 456\, l\, n\, nu + 26\, l^2\, n\, nu -$$
$$26\, m^2\, n\, nu - 456\, mu\, n\, nu - 52\, l\, mu\, n\, nu - 52\, m\, mu\, n\, nu + 1136\, n^2\, nu + 264\, l\, n^2\, nu +$$
$$9\, l^2\, n^2\, nu - 9\, m^2\, n^2\, nu - 264\, mu\, n^2\, nu - 18\, l\, mu\, n^2\, nu - 18\, m\, mu\, n^2\, nu + 414\, n^3\, nu +$$
$$66\, l\, n^3\, nu + l^2\, n^3\, nu - m^2\, n^3\, nu - 66\, mu\, n^3\, nu - 2\, l\, mu\, n^3\, nu - 2\, m\, mu\, n^3\, nu + 73\, n^4\, nu +$$
$$6\, l\, n^4\, nu - 6\, mu\, n^4\, nu + 5\, n^5\, nu + 480\, nu^2 + 96\, l\, nu^2 - 96\, mu\, nu^2 + 712\, n\, nu^2 +$$
$$104\, l\, n\, nu^2 - 104\, mu\, n\, nu^2 + 388\, n^2\, nu^2 + 36\, l\, n^2\, nu^2 - 36\, mu\, n^2\, nu^2 + 92\, n^3\, nu^2 +$$
$$4\, l\, n^3\, nu^2 - 4\, mu\, n^3\, nu^2 + 8\, n^4\, nu^2 + 96\, nu^3 + 104\, n\, nu^3 + 36\, n^2\, nu^3 + 4\, n^3\, nu^3\Big)\; S_n^4 +$$
$$\Big(24 + 24\, l + 24\, m + 24\, l\, m + 68\, n + 68\, l\, n + 68\, m\, n + 68\, l\, m\, n + 68\, n^2 + 68\, l\, n^2 +$$
$$68\, m\, n^2 + 68\, l\, m\, n^2 + 28\, n^3 + 28\, l\, n^3 + 28\, m\, n^3 + 28\, l\, m\, n^3 + 4\, n^4 + 4\, l\, n^4 + 4\, m\, n^4 +$$
$$4\, l\, m\, n^4 + 92\, nu + 92\, l\, nu + 92\, m\, nu + 92\, l\, m\, nu + 180\, n\, nu + 180\, l\, n\, nu + 180\, m\, n\, nu +$$
$$180\, l\, m\, n\, nu + 108\, n^2\, nu + 108\, l\, n^2\, nu + 108\, m\, n^2\, nu + 108\, l\, m\, n^2\, nu + 20\, n^3\, nu +$$
$$20\, l\, n^3\, nu + 20\, m\, n^3\, nu + 20\, l\, m\, n^3\, nu + 112\, nu^2 + 112\, l\, nu^2 + 112\, m\, nu^2 + 112\, l\, m\, nu^2 +$$
$$132\, n\, nu^2 + 132\, l\, n\, nu^2 + 132\, m\, n\, nu^2 + 132\, l\, m\, n\, nu^2 + 36\, n^2\, nu^2 + 36\, l\, n^2\, nu^2 +$$
$$36\, m\, n^2\, nu^2 + 36\, l\, m\, n^2\, nu^2 + 52\, nu^3 + 52\, l\, nu^3 + 52\, m\, nu^3 + 52\, l\, m\, nu^3 + 28\, n\, nu^3 +$$
$$28\, l\, n\, nu^3 + 28\, m\, n\, nu^3 + 28\, l\, m\, n\, nu^3 + 8\, nu^4 + 8\, l\, nu^4 + 8\, m\, nu^4 + 8\, l\, m\, nu^4\Big)\; S_m\, S_n +$$
$$\Big(48 - 72\, l - 24\, l^2 - 24\, m - 24\, l\, m - 144\, mu - 48\, l\, mu + 136\, n - 192\, l\, n - 56\, l^2\, n -$$
$$56\, m\, n - 56\, l\, m\, n - 384\, mu\, n - 112\, l\, mu\, n + 148\, n^2 - 194\, l\, n^2 - 46\, l^2\, n^2 - 46\, m\, n^2 -$$
$$46\, l\, m\, n^2 - 388\, mu\, n^2 - 92\, l\, mu\, n^2 + 78\, n^3 - 94\, l\, n^3 - 16\, l^2\, n^3 - 16\, m\, n^3 - 16\, l\, m\, n^3 -$$
$$188\, mu\, n^3 - 32\, l\, mu\, n^3 + 20\, n^4 - 22\, l\, n^4 - 2\, l^2\, n^4 - 2\, m\, n^4 - 2\, l\, m\, n^4 - 44\, mu\, n^4 -$$
$$4\, l\, mu\, n^4 + 2\, n^5 - 2\, l\, n^5 - 4\, mu\, n^5 + 216\, nu - 140\, l\, nu - 36\, l^2\, nu - 68\, m\, nu - 68\, l\, m\, nu +$$
$$16\, m^2\, nu - 376\, mu\, nu - 104\, l\, mu\, nu + 32\, m\, mu\, nu + 496\, n\, nu - 294\, l\, n\, nu - 62\, l^2\, n\, nu -$$
$$102\, m\, n\, nu - 102\, l\, m\, n\, nu + 24\, m^2\, n\, nu - 748\, mu\, n\, nu - 156\, l\, mu\, n\, nu + 48\, m\, mu\, n\, nu +$$
$$430\, n^2\, nu - 220\, l\, n^2\, nu - 34\, l^2\, n^2\, nu - 50\, m\, n^2\, nu - 50\, l\, m\, n^2\, nu + 12\, m^2\, n^2\, nu -$$
$$544\, mu\, n^2\, nu - 76\, l\, mu\, n^2\, nu + 24\, m\, mu\, n^2\, nu + 174\, n^3\, nu - 70\, l\, n^3\, nu - 6\, l^2\, n^3\, nu -$$
$$8\, m\, n^3\, nu - 8\, l\, m\, n^3\, nu + 2\, m^2\, n^3\, nu - 172\, mu\, n^3\, nu - 12\, l\, mu\, n^3\, nu + 4\, m\, mu\, n^3\, nu +$$
$$32\, n^4\, nu - 8\, l\, n^4\, nu - 20\, mu\, n^4\, nu + 2\, n^5\, nu + 304\, nu^2 - 132\, l\, nu^2 - 28\, l^2\, nu^2 -$$
$$44\, m\, nu^2 - 44\, l\, m\, nu^2 + 24\, m^2\, nu^2 - 280\, mu\, nu^2 - 40\, l\, mu\, nu^2 + 48\, m\, mu\, nu^2 + 556\, n\, nu^2 -$$
$$194\, l\, n\, nu^2 - 30\, l^2\, n\, nu^2 - 42\, m\, n\, nu^2 - 42\, l\, m\, n\, nu^2 + 24\, m^2\, n\, nu^2 - 412\, mu\, n\, nu^2 -$$
$$36\, l\, mu\, n\, nu^2 + 48\, m\, mu\, n\, nu^2 + 366\, n^2\, nu^2 - 92\, l\, n^2\, nu^2 - 8\, l^2\, n^2\, nu^2 - 10\, m\, n^2\, nu^2 -$$
$$10\, l\, m\, n^2\, nu^2 + 6\, m^2\, n^2\, nu^2 - 200\, mu\, n^2\, nu^2 - 8\, l\, mu\, n^2\, nu^2 + 12\, m\, mu\, n^2\, nu^2 + 102\, n^3\, nu^2 -$$
$$14\, l\, n^3\, nu^2 - 32\, mu\, n^3\, nu^2 + 10\, n^4\, nu^2 + 168\, nu^3 - 72\, l\, nu^3 - 8\, l^2\, nu^3 - 8\, m\, nu^3 -$$
$$8\, l\, m\, nu^3 + 8\, m^2\, nu^3 - 64\, mu\, nu^3 + 16\, m\, mu\, nu^3 + 236\, n\, nu^3 - 68\, l\, n\, nu^3 - 4\, l^2\, n\, nu^3 -$$
$$4\, m\, n\, nu^3 - 4\, l\, m\, n\, nu^3 + 4\, m^2\, n\, nu^3 - 64\, mu\, n\, nu^3 + 8\, m\, mu\, n\, nu^3 + 108\, n^2\, nu^3 - 16\, l\, n^2\, nu^3 -$$
$$16\, mu\, n^2\, nu^3 + 16\, n^3\, nu^3 + 32\, nu^4 - 16\, l\, nu^4 + 32\, n\, nu^4 - 8\, l\, n\, nu^4 + 8\, n^2\, nu^4\Big)\; S_n^2 +$$
$$\Big(12\, l\, n + 6\, l^2\, n - 12\, m\, n - 12\, l\, m\, n + 6\, m^2\, n - 24\, mu\, n - 12\, l\, mu\, n + 12\, m\, mu\, n - 12\, n^2 +$$
$$22\, l\, n^2 + 11\, l^2\, n^2 - 22\, m\, n^2 - 22\, l\, m\, n^2 + 11\, m^2\, n^2 - 56\, mu\, n^2 - 22\, l\, mu\, n^2 + 22\, m\, mu\, n^2 -$$
$$28\, n^3 + 12\, l\, n^3 + 6\, l^2\, n^3 - 12\, m\, n^3 - 12\, l\, m\, n^3 + 6\, m^2\, n^3 - 46\, mu\, n^3 - 12\, l\, mu\, n^3 + 12\, m\, mu\, n^3 -$$
$$23\, n^4 + 2\, l\, n^4 + l^2\, n^4 - 2\, m\, n^4 - 2\, l\, m\, n^4 + m^2\, n^4 - 16\, mu\, n^4 - 2\, l\, mu\, n^4 + 2\, m\, mu\, n^4 - 8\, n^5 -$$
$$2\, mu\, n^5 - n^6 + 24\, l\, nu + 12\, l^2\, nu - 24\, m\, nu - 24\, l\, m\, nu + 12\, m^2\, nu - 48\, mu\, nu - 24\, l\, mu\, nu +$$
$$24\, m\, mu\, nu - 24\, n\, nu + 78\, l\, n\, nu + 36\, l^2\, n\, nu - 78\, m\, n\, nu - 78\, l\, m\, n\, nu + 42\, m^2\, n\, nu -$$
$$180\, mu\, n\, nu - 72\, l\, mu\, n\, nu + 84\, m\, mu\, n\, nu - 90\, n^2\, nu + 64\, l\, n^2\, nu + 25\, l^2\, n^2\, nu - 58\, m\, n^2\, nu -$$
$$58\, l\, m\, n^2\, nu + 33\, m^2\, n^2\, nu - 200\, mu\, n^2\, nu - 50\, l\, mu\, n^2\, nu + 66\, m\, mu\, n^2\, nu - 100\, n^3\, nu +$$

$20\, l\, n^3\, nu + 5\, l^2\, n^3\, nu - 12\, m\, n^3\, nu - 12\, l\, m\, n^3\, nu + 7\, m^2\, n^3\, nu - 90\, mu\, n^3\, nu - 10\, l\, mu\, n^3\, nu +$
$14\, m\, mu\, n^3\, nu - 45\, n^4\, nu + 2\, l\, n^4\, nu - 14\, mu\, n^4\, nu - 7\, n^5\, nu + 68\, l\, nu^2 + 28\, l^2\, nu^2 -$
$68\, m\, nu^2 - 68\, l\, m\, nu^2 + 40\, m^2\, nu^2 - 136\, mu\, nu^2 - 56\, l\, mu\, nu^2 + 80\, m\, mu\, nu^2 - 68\, n\, nu^2 +$
$102\, l\, n\, nu^2 + 30\, l^2\, n\, nu^2 - 90\, m\, n\, nu^2 - 90\, l\, m\, n\, nu^2 + 60\, m^2\, n\, nu^2 - 260\, mu\, n\, nu^2 -$
$60\, l\, mu\, n\, nu^2 + 120\, m\, mu\, n\, nu^2 - 130\, n^2\, nu^2 + 56\, l\, n^2\, nu^2 + 8\, l^2\, n^2\, nu^2 - 26\, m\, n^2\, nu^2 -$
$26\, l\, m\, n^2\, nu^2 + 18\, m^2\, n^2\, nu^2 - 172\, mu\, n^2\, nu^2 - 16\, l\, mu\, n^2\, nu^2 + 36\, m\, mu\, n^2\, nu^2 - 86\, n^3\, nu^2 +$
$10\, l\, n^3\, nu^2 - 36\, mu\, n^3\, nu^2 - 18\, n^4\, nu^2 + 44\, l\, nu^3 + 8\, l^2\, nu^3 - 44\, m\, nu^3 - 44\, l\, m\, nu^3 +$
$36\, m^2\, nu^3 - 88\, mu\, nu^3 - 16\, l\, mu\, nu^3 + 72\, m\, mu\, nu^3 - 44\, n\, nu^3 + 52\, l\, n\, nu^3 + 4\, l^2\, n\, nu^3 -$
$24\, m\, n\, nu^3 - 24\, l\, m\, n\, nu^3 + 20\, m^2\, n\, nu^3 - 120\, mu\, n\, nu^3 - 8\, l\, mu\, n\, nu^3 + 40\, m\, mu\, n\, nu^3 -$
$60\, n^2\, nu^3 + 16\, l\, n^2\, nu^3 - 40\, mu\, n^2\, nu^3 - 20\, n^3\, nu^3 + 8\, l\, nu^4 - 8\, m\, nu^4 - 8\, l\, m\, nu^4 +$
$8\, m^2\, nu^4 - 16\, mu\, nu^4 + 16\, m\, mu\, nu^4 - 8\, n\, nu^4 + 8\, l\, n\, nu^4 - 16\, mu\, n\, nu^4 - 8\, n^2\, nu^4) \},$

$\{\{0\}, \{0\}, \{2\, (1 + m + mu + 2\, n + 2\, m\, n + 2\, mu\, n + n^2 + m\, n^2 + mu\, n^2 + nu + m\, nu + mu\, nu +$
$n\, nu + m\, n\, nu + mu\, n\, nu)\, x\, S_m\, S_n - 2\, (n + m\, n + mu\, n + n^2 + m\, n^2 + mu\, n^2 + 2\, nu +$
$2\, m\, nu + 2\, mu\, nu + 3\, n\, nu + 3\, m\, n\, nu + 3\, mu\, n\, nu + 2\, nu^2 + 2\, m\, nu^2 + 2\, mu\, nu^2)\, x^2\, S_m -$
$2\, (1 + m + mu + 2\, n + 2\, m\, n + 2\, mu\, n + n^2 + m\, n^2 + mu\, n^2 + nu + m\, nu + mu\, nu + n\, nu +$
$m\, n\, nu + mu\, n\, nu)\, x^2\, S_n + 2\, (n + m\, n + mu\, n + n^2 + m\, n^2 + mu\, n^2 + 2\, nu + 2\, m\, nu +$
$2\, mu\, nu + 3\, n\, nu + 3\, m\, n\, nu + 3\, mu\, n\, nu + 2\, nu^2 + 2\, m\, nu^2 + 2\, mu\, nu^2)\, x\},$

$\{-2\, (2 + 5\, n + 4\, n^2 + n^3 + 3\, nu + 5\, n\, nu + 2\, n^2\, nu + nu^2 + n\, nu^2)\, x^2\, S_m\, S_n +$
$2\, (2\, n + 3\, n^2 + n^3 + 4\, nu + 9\, n\, nu + 4\, n^2\, nu + 6\, nu^2 + 5\, n\, nu^2 + 2\, nu^3)\, x\, S_m +$
$2\, (-2\, x - 5\, n\, x - 4\, n^2\, x - n^3\, x - 7\, nu\, x - 11\, n\, nu\, x - 4\, n^2\, nu\, x - 7\, nu^2\, x -$
$5\, n\, nu^2\, x - 2\, nu^3\, x + 4\, x^3 + 10\, n\, x^3 + 8\, n^2\, x^3 + 2\, n^3\, x^3 + 10\, nu\, x^3 +$
$16\, n\, nu\, x^3 + 6\, n^2\, nu\, x^3 + 8\, nu^2\, x^3 + 6\, n\, nu^2\, x^3 + 2\, nu^3\, x^3)\, S_n -$
$2\, (2\, n + 3\, n^2 + n^3 + 4\, nu + 9\, n\, nu + 4\, n^2\, nu + 6\, nu^2 + 5\, n\, nu^2 + 2\, nu^3)\, x^2\},$

$\{4\, (-6\, x - 6\, m\, x - 17\, n\, x - 17\, m\, n\, x - 17\, n^2\, x - 17\, m\, n^2\, x - 7\, n^3\, x - 7\, m\, n^3\, x - n^4\, x -$
$m\, n^4\, x - 23\, nu\, x - 23\, m\, nu\, x - 45\, n\, nu\, x - 45\, m\, n\, nu\, x - 27\, n^2\, nu\, x - 27\, m\, n^2\, nu\, x -$
$5\, n^3\, nu\, x - 5\, m\, n^3\, nu\, x - 28\, nu^2\, x - 28\, m\, nu^2\, x - 33\, n\, nu^2\, x - 33\, m\, n\, nu^2\, x -$
$9\, n^2\, nu^2\, x - 9\, m\, n^2\, nu^2\, x - 13\, nu^3\, x - 13\, m\, nu^3\, x - 7\, n\, nu^3\, x - 7\, m\, n\, nu^3\, x -$
$2\, nu^4\, x - 2\, m\, nu^4\, x + 12\, x^3 + 12\, m\, x^3 + 34\, n\, x^3 + 34\, m\, n\, x^3 + 34\, n^2\, x^3 + 34\, m\, n^2\, x^3 +$
$14\, n^3\, x^3 + 14\, m\, n^3\, x^3 + 2\, n^4\, x^3 + 2\, m\, n^4\, x^3 + 34\, nu\, x^3 + 34\, m\, nu\, x^3 + 68\, n\, nu\, x^3 +$
$68\, m\, n\, nu\, x^3 + 42\, n^2\, nu\, x^3 + 42\, m\, n^2\, nu\, x^3 + 8\, n^3\, nu\, x^3 + 8\, m\, n^3\, nu\, x^3 + 34\, nu^2\, x^3 +$
$34\, m\, nu^2\, x^3 + 42\, n\, nu^2\, x^3 + 42\, m\, n\, nu^2\, x^3 + 12\, n^2\, nu^2\, x^3 + 12\, m\, n^2\, nu^2\, x^3 +$
$14\, nu^3\, x^3 + 14\, m\, nu^3\, x^3 + 8\, n\, nu^3\, x^3 + 8\, m\, n\, nu^3\, x^3 + 2\, nu^4\, x^3 + 2\, m\, nu^4\, x^3)\, S_m\, S_n -$
$4\, (6\, n + 6\, m\, n + 11\, n^2 + 11\, m\, n^2 + 6\, n^3 + 6\, m\, n^3 + n^4 + m\, n^4 + 12\, nu + 12\, m\, nu +$
$33\, n\, nu + 33\, m\, n\, nu + 24\, n^2\, nu + 24\, m\, n^2\, nu + 5\, n^3\, nu + 5\, m\, n^3\, nu +$
$22\, nu^2 + 22\, m\, nu^2 + 30\, n\, nu^2 + 30\, m\, n\, nu^2 + 9\, n^2\, nu^2 + 9\, m\, n^2\, nu^2 +$
$12\, nu^3 + 12\, m\, nu^3 + 7\, n\, nu^3 + 7\, m\, n\, nu^3 + 2\, nu^4 + 2\, m\, nu^4)\, x^2\, S_m -$
$4\, (-42\, x^2 - 12\, l\, x^2 - 6\, m\, x^2 - 131\, n\, x^2 - 34\, l\, n\, x^2 - 17\, m\, n\, x^2 - 153\, n^2\, x^2 - 34\, l\, n^2\, x^2 -$
$17\, m\, n^2\, x^2 - 83\, n^3\, x^2 - 14\, l\, n^3\, x^2 - 7\, m\, n^3\, x^2 - 21\, n^4\, x^2 - 2\, l\, n^4\, x^2 - m\, n^4\, x^2 - 2\, n^5\, x^2 -$
$149\, nu\, x^2 - 34\, l\, nu\, x^2 - 23\, m\, nu\, x^2 - 351\, n\, nu\, x^2 - 68\, l\, n\, nu\, x^2 - 45\, m\, n\, nu\, x^2 -$
$289\, n^2\, nu\, x^2 - 42\, l\, n^2\, nu\, x^2 - 27\, m\, n^2\, nu\, x^2 - 99\, n^3\, nu\, x^2 - 8\, l\, n^3\, nu\, x^2 - 5\, m\, n^3\, nu\, x^2 -$
$12\, n^4\, nu\, x^2 - 198\, nu^2\, x^2 - 34\, l\, nu^2\, x^2 - 28\, m\, nu^2\, x^2 - 329\, n\, nu^2\, x^2 - 42\, l\, n\, nu^2\, x^2 -$
$33\, m\, n\, nu^2\, x^2 - 171\, n^2\, nu^2\, x^2 - 12\, l\, n^2\, nu^2\, x^2 - 9\, m\, n^2\, nu^2\, x^2 - 28\, n^3\, nu^2\, x^2 -$
$123\, nu^3\, x^2 - 14\, l\, nu^3\, x^2 - 13\, m\, nu^3\, x^2 - 129\, n\, nu^3\, x^2 - 8\, l\, n\, nu^3\, x^2 - 7\, m\, n\, nu^3\, x^2 -$
$32\, n^2\, nu^3\, x^2 - 36\, nu^4\, x^2 - 2\, l\, nu^4\, x^2 - 2\, m\, nu^4\, x^2 - 18\, n\, nu^4\, x^2 - 4\, nu^5\, x^2 + 48\, x^4 +$
$12\, l\, x^4 + 12\, m\, x^4 + 148\, n\, x^4 + 34\, l\, n\, x^4 + 34\, m\, n\, x^4 + 170\, n^2\, x^4 + 34\, l\, n^2\, x^4 + 34\, m\, n^2\, x^4 +$

$$90 n^3 x^4 + 14 l n^3 x^4 + 14 m n^3 x^4 + 22 n^4 x^4 + 2 l n^4 x^4 + 2 m n^4 x^4 + 2 n^5 x^4 + 160 nu x^4 + 34 l nu x^4 + 34 m nu x^4 + 374 n nu x^4 + 68 l n nu x^4 + 68 m n nu x^4 + 304 n^2 nu x^4 + 42 l n^2 nu x^4 + 42 m n^2 nu x^4 + 102 n^3 nu x^4 + 8 l n^3 nu x^4 + 8 m n^3 nu x^4 + 12 n^4 nu x^4 + 204 nu^2 x^4 + 34 l nu^2 x^4 + 34 m nu^2 x^4 + 338 n nu^2 x^4 + 42 l n nu^2 x^4 + 42 m n nu^2 x^4 + 174 n^2 nu^2 x^4 + 12 l n^2 nu^2 x^4 + 12 m n^2 nu^2 x^4 + 28 n^3 nu^2 x^4 + 124 nu^3 x^4 + 14 l nu^3 x^4 + 14 m nu^3 x^4 + 130 n nu^3 x^4 + 8 l n nu^3 x^4 + 8 m n nu^3 x^4 + 32 n^2 nu^3 x^4 + 36 nu^4 x^4 + 2 l nu^4 x^4 + 2 m nu^4 x^4 + 18 n nu^4 x^4 + 4 nu^5 x^4) S_n +$$

$$4 (-18 n x - 6 l n x - 39 n^2 x - 11 l n^2 x - 29 n^3 x - 6 l n^3 x - 9 n^4 x - l n^4 x - n^5 x - 36 nu x - 12 l nu x - 123 n nu x - 33 l n nu x - 127 n^2 nu x - 24 l n^2 nu x - 51 n^3 nu x - 5 l n^3 nu x - 7 n^4 nu x - 90 nu^2 x - 22 l nu^2 x - 178 n nu^2 x - 30 l n nu^2 x - 105 n^2 nu^2 x - 9 l n^2 nu^2 x - 19 n^3 nu^2 x - 80 nu^3 x - 12 l nu^3 x - 93 n nu^3 x - 7 l n nu^3 x - 25 n^2 nu^3 x - 30 nu^4 x - 2 l nu^4 x - 16 n nu^4 x - 4 nu^5 x + 24 n x^3 + 6 l n x^3 + 6 m n x^3 + 50 n^2 x^3 + 11 l n^2 x^3 + 11 m n^2 x^3 + 35 n^3 x^3 + 6 l n^3 x^3 + 6 m n^3 x^3 + 10 n^4 x^3 + l n^4 x^3 + m n^4 x^3 + n^5 x^3 + 48 nu x^3 + 12 l nu x^3 + 12 m nu x^3 + 156 n nu x^3 + 33 l n nu x^3 + 33 m n nu x^3 + 151 n^2 nu x^3 + 24 l n^2 nu x^3 + 24 m n^2 nu x^3 + 56 n^3 nu x^3 + 5 l n^3 nu x^3 + 5 m n^3 nu x^3 + 7 n^4 nu x^3 + 112 nu^2 x^3 + 22 l nu^2 x^3 + 22 m nu^2 x^3 + 208 n nu^2 x^3 + 30 l n nu^2 x^3 + 30 m n nu^2 x^3 + 114 n^2 nu^2 x^3 + 9 l n^2 nu^2 x^3 + 9 m n^2 nu^2 x^3 + 19 n^3 nu^2 x^3 + 92 nu^3 x^3 + 12 l nu^3 x^3 + 12 m nu^3 x^3 + 100 n nu^3 x^3 + 7 l n nu^3 x^3 + 7 m n nu^3 x^3 + 25 n^2 nu^3 x^3 + 32 nu^4 x^3 + 2 l nu^4 x^3 + 2 m nu^4 x^3 + 16 n nu^4 x^3 + 4 nu^5 x^3) \} \} \} \}$$