EFFICIENT COMPUTATION OF POWER SERIES COEFFICIENTS

Field: Computer algebra
Institution: INRIA - Saclay-ÎDF
Place: Palaiseau (France)
Team: SpecFun
Internship supervisors:
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General presentation of the field:

The need to compute a single coefficient of a given formal power series is frequent in combinatorics. Indeed, in many case, the enumeration of objects subject to certain structural constraints is reduced to extracting a coefficient of a series. For instance, the number of undirected graphs with n vertices and exactly 2 edges from each vertex is equal to $n!$ times the coefficient of z^n in the series expansion at the origin of $(1-z)^{-1/2}\exp(-z/2-z^2/4)$. In more elaborate contexts, several parameters are involved, and the corresponding power series have several variables. The question is then to extract the coefficient of a monomial in several variables. For example, the number of partitions of a set with n elements in k parts is equal to $n!$ times the coefficient of $w^k z^n$ in the expansion at the origin of the bivariate series $\exp\big(w(e^z-1)\big)$. For obvious efficiency reasons, it is important to be able to calculate this coefficient without computing all previous coefficients.

Goals of the internship:

This internship focuses on fast algorithms for extracting such coefficients for classes of power series whose coefficients are related by a set of linear recurrences. The latter are then used to calculate all the coefficients step by step. The naive calculation of the coefficient of index $(N_1,\dots,N_r)$ amounts to first calculating essentially all coefficients of indices $(n_1,\dots,n_r)$ for $n_1+\cdots+n_r < N = N_1+\cdots+N_r$. The calculation of a single coefficient therefore requires a number of operations of order N^r, making calculations intractable in practice.

In the articles cited below, Massazza and Radicioni proposed two algorithms for calculating a single coefficient in a number of operations of the order of only N for two classes of series, namely for series that come from rational functions and series satisfying a system of linear differential equations. The first objective of the internship is to implement these algorithms and compare them on various applications.

If time permits, we will then look to further accelerate these algorithms, as conventional techniques (Bostan et al., 2014) seem to apply.

Bibliography:

- P. Massazza and R. Radicioni: On computing the coefficients of rational formal series. (2004). In: *Formal Power Series and Algebraic Combinatorics, Vancouver, 2004*.
- P. Massazza and R. Radicioni: On computing the coefficients of bivariate holonomic formal series. *Theoretical Computer Science* 346 (2005), no. 2-3, 418–438.
- A. Bostan, F. Chyzak, M. Giusti, R. Lebreton, B. Salvy, É. Schost: Algorithmes Efficaces en Calcul Formel. Notes du cours 2-22 du MPRI (2014), http://perso.ens-lyon.fr/bruno.salvy/mpri/poly.pdf.

Required skills:

The intern should have the taste for computer programming on mathematical objects. A minimal knowledge of a general-purpose computer algebra system would be a plus.