EFFICIENT SYMBOLIC RESOLUTION OF SYSTEMS OF LINEAR DIFFERENTIAL AND DIFFERENCE EQUATIONS

Field: Computer algebra
Institution: INRIA - Saclay-ÎDF
Place: Palaiseau (France)
Team: SpecFun
Internship supervisors:
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General presentation of the field:
In the context of problems involving solutions of linear differential or difference equations, computer algebra is often crucially based on the resolution of auxiliary systems in terms of rational functions. This is indeed the bottleneck of algorithms for the symbolic resolution of equations in terms of special functions, for factoring operators, for symbolic summation and integration, etc. The simplest approach to this problem is to uncouple the system, i.e. to obtain a scalar equation of higher order for each component of the vector solutions. Rational solutions are the computed using algorithms due to Abramov for the resolution of scalar equations. The uncoupling procedure by itself is already time consuming, so it seems advantageous to use direct solution algorithms that avoid uncoupling. The basis of such an algorithm exists (Barkatou, 1999), but it remains to adapt it to applications and to precisely quantify the expected gain.

Goals of the internship: The list of goals below more or less follows a natural order of dependency and an order of increasing difficulty. The internship is expected to complete the first 2 points and to include at least one or two of the following, depending on the tastes of the intern and on the difficulties encountered:

- First, the algorithm described in (Barkatou, 1999) will be validated by an implementation in Maple, first in its simplest version, then in its parameterized version.
- This parameterized version will then be integrated in the `Mgfun` package that implements algorithms by (Chyzak, 2000) for the symbolic summation and integration in Maple.
- Systems obtained in applications are often sparse (they have many zero coefficients). We will study the algorithmic impact of the sparse shape of the input system, and we will try to exploit this sparsity in order to improve the algorithm.
- The theoretical complexity of Barkatou's algorithm will be studied.
- Barkatou's algorithm will be compared with the cyclic vector approach, for which a complexity analysis was performed in (Bostan, Chyzak, Panafieu, 2013).

Bibliography:

- Moulay A. Barkatou: On rational solutions of systems of linear differential equations. *Journal of Symbolic Computation* 28 (1999), 547–567.
- Alin Bostan, Frédéric Chyzak, Élie de Panafieu: Complexity estimates for two uncoupling algorithms. In: *ISSAC'13, 38th International Symposium on Symbolic and Algebraic Computation*, July 2013, Boston, United States, 85–92.
- Frédéric Chyzak: An extension of Zeilberger's fast algorithm to general holonomic functions. *Discrete Mathematics* 217 (2000), 115–134.

Required skills:
The intern should have the taste for computer programming on mathematical objects. A minimal knowledge of a general-purpose computer algebra system would be a plus.