ALGORITHMIC PROOFS FOR THE TRANSCENDENCE OF POWER SERIES

Field: Computer algebra
Institution: INRIA - Saclay-ÎDF
Place: Palaiseau (France)
Team: SpecFun
Internship supervisors:
Alin Bostan (alin.bostan((at))inria.fr) and
Frédéric Chyzak (frederic.chyzak((at))inria.fr)
Lab director: Nozha Boujemaa (nozha.boujemaa((at))inria.fr)

General presentation of the field: A formal power series $\sum_{n\geq 0} a_n t^n$ in $\mathbb{Q}[[t]]$ is called algebraic if it is a root of a polynomial $P(t,s) \in \mathbb{Q}[t,s]$, and transcendental, otherwise.

A constant need in enumerative combinatorics is to establish the algebraicity, or the transcendence, of a generating function; see (Bousquet-Mélou, 2005). For instance, it is known that the counting sequence for planar maps with n edges admits an algebraic generating function, and that the generating series of walks of a chess king in the quarter plane is transcendental. In both cases, the counting sequence is P-recursive, that is, it satisfies a linear recurrence with polynomial coefficients. Several approaches exist for proving the transcendence of the generating series of such a sequence. For example, asymptotic information (Flajolet, 1987), or arithmetic information (Allouche, 1999), on the counting sequence may help to conclude in certain cases.

Goals of the internship: The goals of the internship are the study, the comparison and the implementation of algorithmic methods allowing to answer in a uniform manner the following question:

> *Given a sequence $(a_n)_{n\geq 0}$ represented by a linear recurrence with polynomial coefficients and sufficient initial terms, decide the transcendence of the power series $\sum_{n\geq 0} a_n t^n$.*

The question is non trivial even for sequences satisfying a recurrence of first order. A classic algorithm is sufficient, in principle, to answer the general case (Singer, 1979). However, this algorithm suffers from too high a complexity to be effective in practice. A more recent method has been proposed, on a non-trivial combinatorial example, in (Bostan et al, 2014). It helps to reduce the question of transcendence to a linear algebra question. The central objectives of this internship are to understand the new method, to translate it into an explicit algorithm working uniformly on a class of inputs to be defined, to implement this algorithm, test it and measure its theoretical and practical efficiency. Ideally, the internship will result in a hybrid algorithm combining the new method, and the asymptotic, arithmetic and algebraic methods mentioned above.

Bibliography:

- J.-P. Allouche: Transcendence of formal power series with rational coefficients. *Theoretical Computer Science* 218 (1999), 143–160.
- A. Bostan, M. Bousquet-Mélou, M. Kauers, S. Melczer : On 3-dimensional lattice walks confined to the positive octant (2014), http://arxiv.org/abs/1409.3669.
- M. Bousquet-Mélou: Algebraic generating functions in enumerative combinatorics, and context-free languages. *STACS 2005, Lect. Notes Comput. Sci.* 3404 (2005), 18–35.
- Ph. Flajolet: Analytic models and ambiguity of context-free languages. *Theoretical Computer Science* 49 (1987), 283–309.
- M. F. Singer: Algebraic Solutions of nth Order Linear Differential Equations. *Queens Papers in Pure and Applied Mathematics*, 54 (1979), 379–420.

Required skills:

The intern should have the taste for computer programming on mathematical objects. A minimal knowledge of a general-purpose computer algebra system would be a plus.